New exact solutions of Navier-Stokes equations

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Introduction:

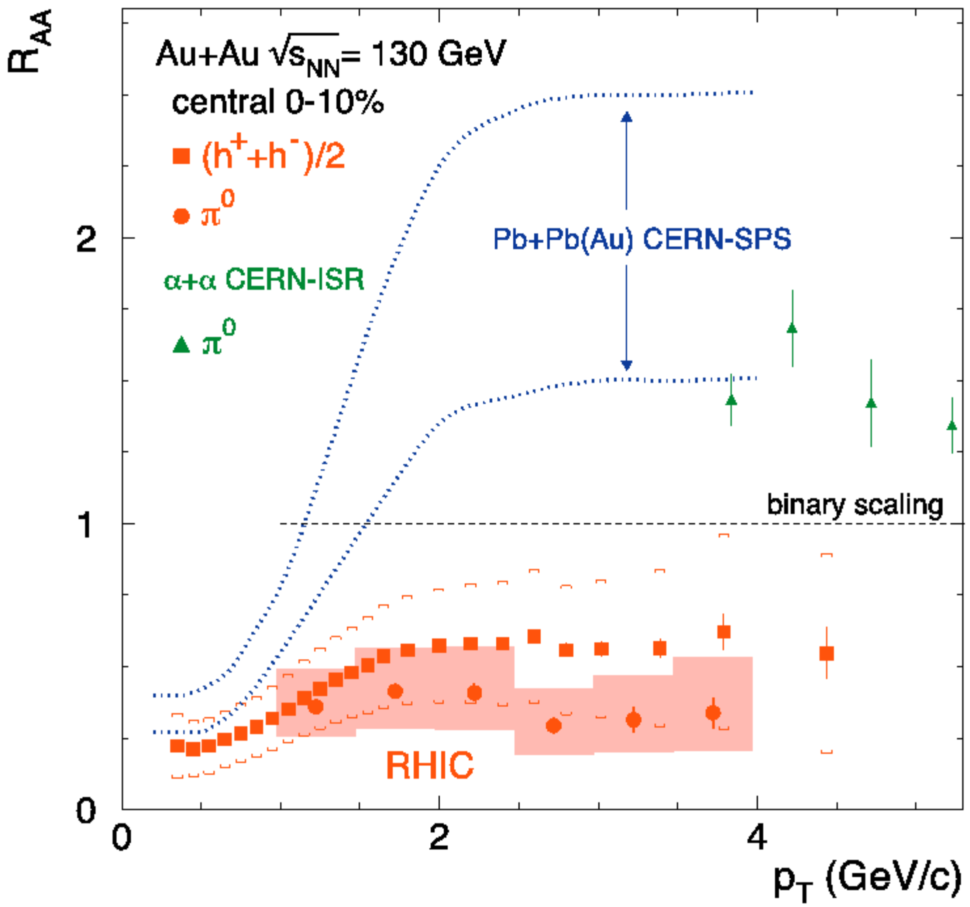
- **"RHIC Serves the Perfect Liquid", BNL Press Release, 2005 IV. 18**
- **BRAHMS, PHENIX, PHOBOS, STAR White Papers in NPA, 2005**
- **2005 AIP top physics story, 2006 "silver medal" nucl-ex paper**

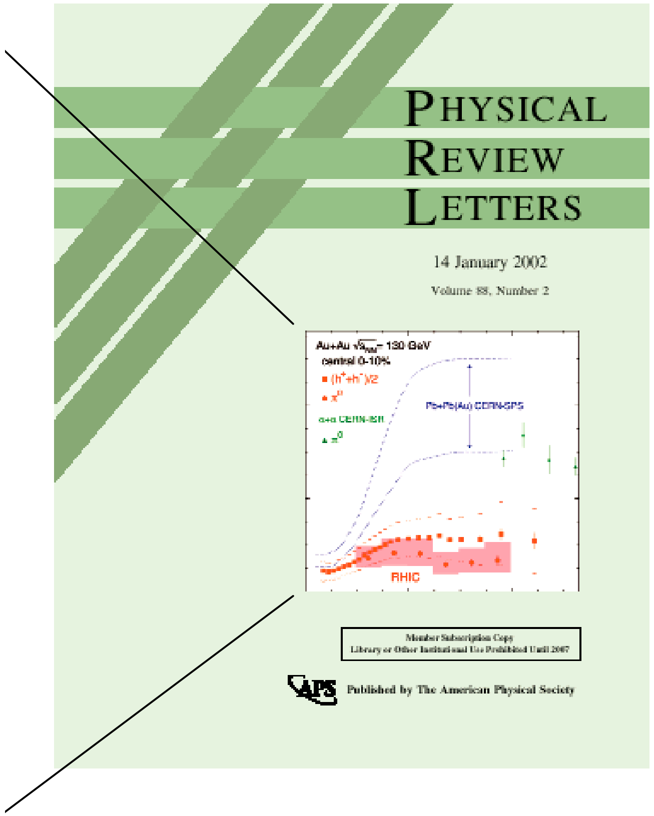
Indication of hydro in RHIC/SPS data: hydrodynamical scaling behavior

- **Appear in beautiful, exact family of solutions of fireball hydro**
  - **non-relativistic, perfect and dissipative exact solutions**
  - **relativistic, perfect, accelerating solutions -> M. Nagy's talk**
- **Their application to data analysis at RHIC energies -> Buda-Lund**

Exact results: tell us what can and what cannot be learned from data

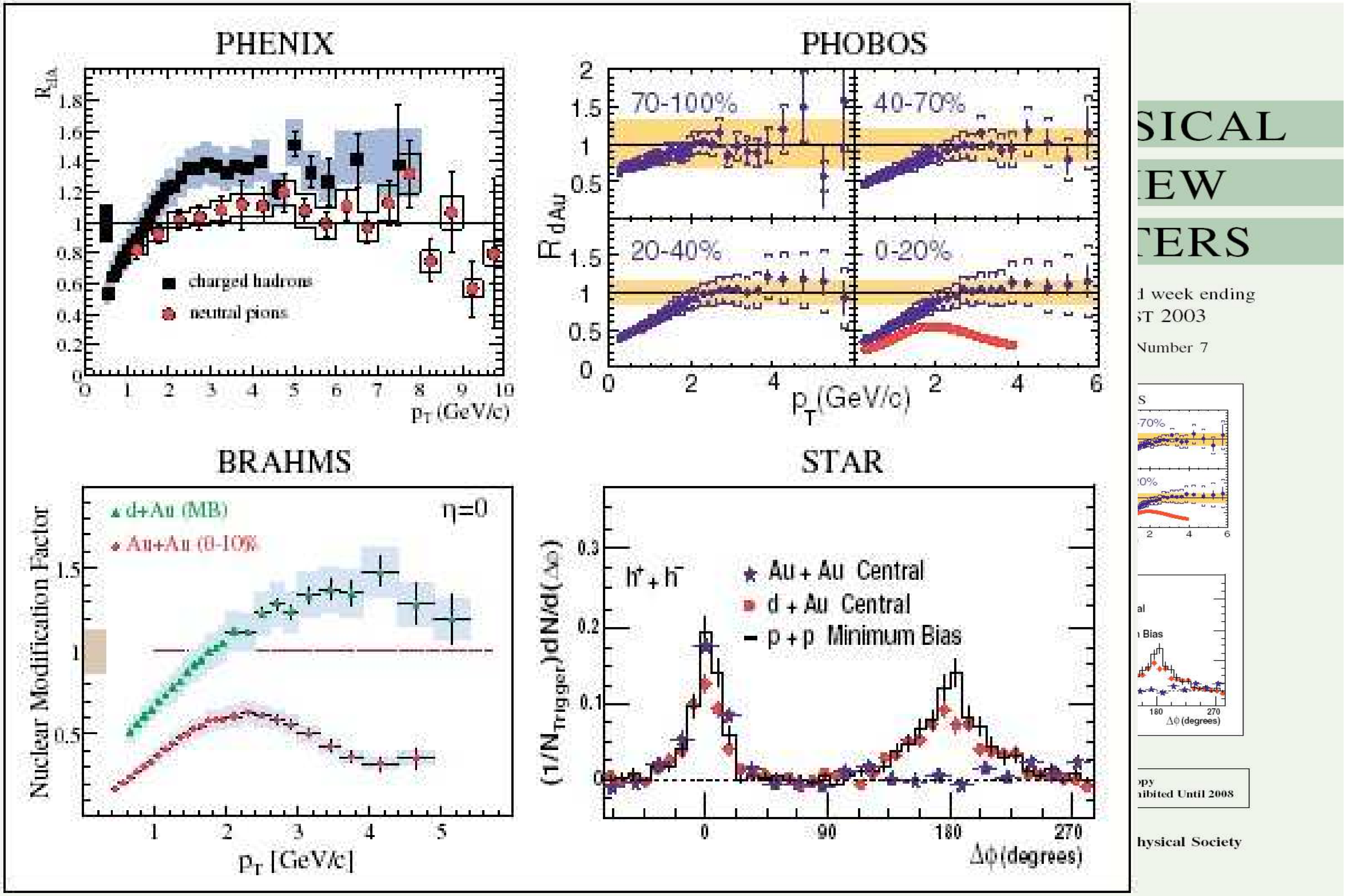
1st milestone: new phenomena



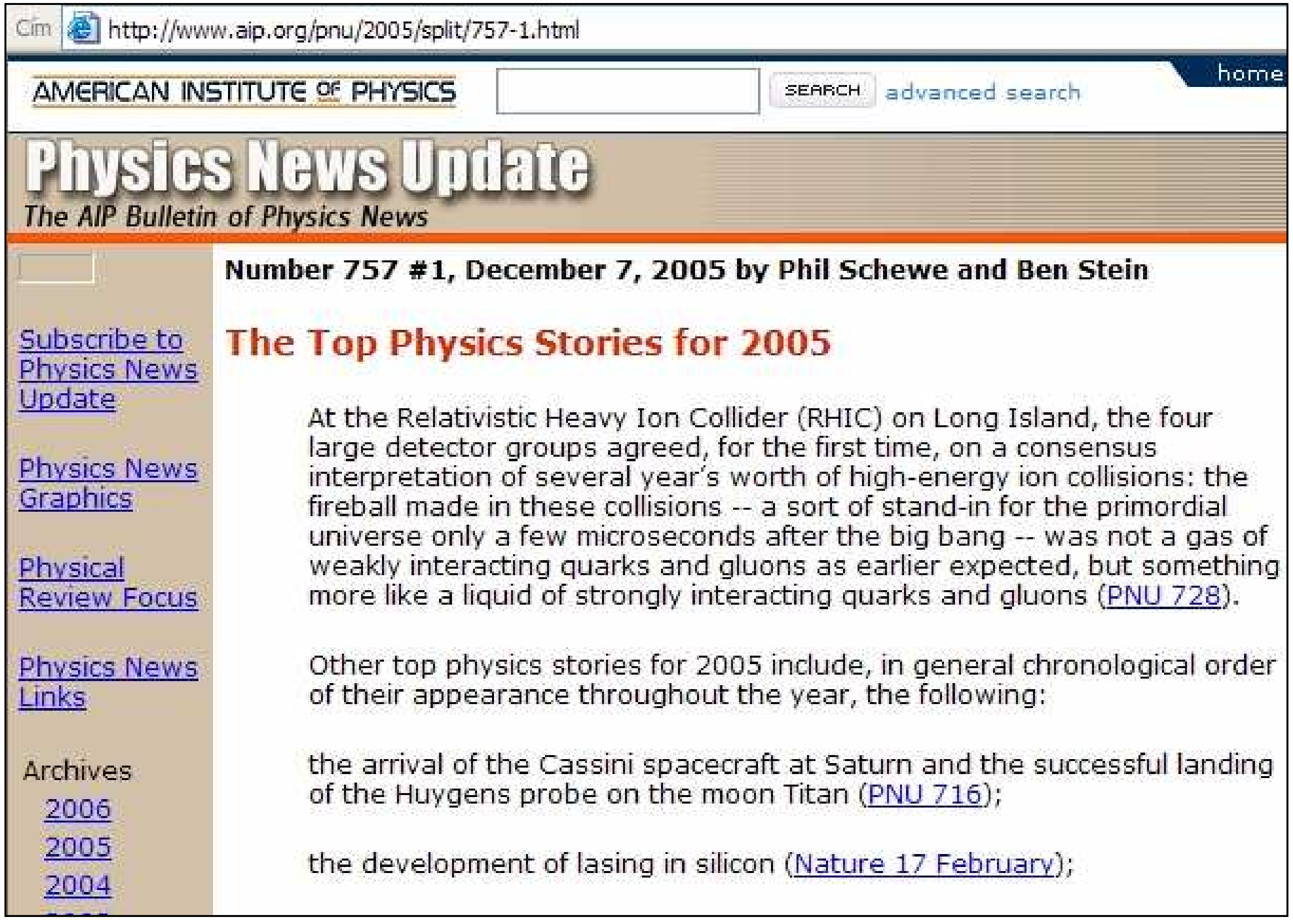


Suppression of high p_t particle production in Au+Au collisions at RHIC

2nd milestone: new form of matter



3rd milestone: Top Physics Story 2005



Cím http://www.aip.org/pnu/2005/split/757-1.html

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Number 757 #1, December 7, 2005 by Phil Schewe and Ben Stein

The Top Physics Stories for 2005

At the Relativistic Heavy Ion Collider (RHIC) on Long Island, the four large detector groups agreed, for the first time, on a consensus interpretation of several year's worth of high-energy ion collisions: the fireball made in these collisions -- a sort of stand-in for the primordial universe only a few microseconds after the big bang -- was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons (PNU 728).

Other top physics stories for 2005 include, in general chronological order of their appearance throughout the year, the following:

the arrival of the Cassini spacecraft at Saturn and the successful landing of the Huygens probe on the moon Titan (PNU 716);

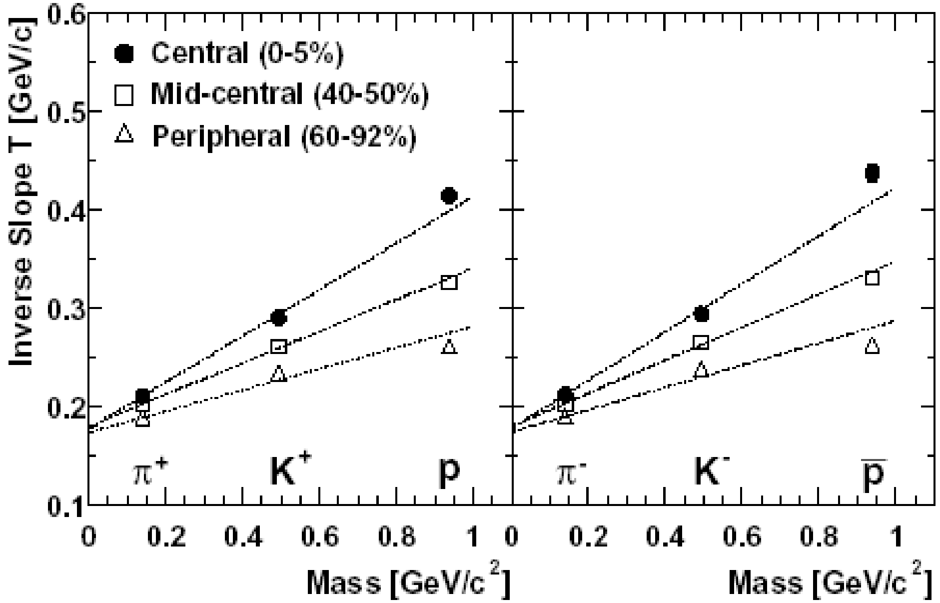
the development of lasing in silicon (Nature 17 February);

http://arxiv.org/abs/nucl-ex/0410003

PHENIX White Paper: second most cited in nucl-ex during 2006

An observation:

PHENIX, Phys. Rev. C69, 034909 (2004)



Inverse slopes T of single particle p_t distribution
increase linearly with mass:

$$T = T_0 + m\langle u_t\rangle^2$$

Increase is stronger in more head-on collisions.
Suggests collective radial flow, local thermalization and hydrodynamics
Nu Xu, NA44 collaboration, Pb+Pb @ CERN SPS

Notation for fluid dynamics

- **nonrelativistic hydro:**

  t: time,

  r: coordinate 3-vector, $r = (r_x, r_y, r_z)$,

  m: mass,

- **field i.e. (t,r) dependent variables:**

  n: number density,

  σ: entropy density,

  p: pressure,

  ε: energy density,

  T: temperature,

  v: velocity 3-vector, $v = (v_x, v_y, v_z)$,

- **relativistic hydro:**

  x^μ: coordinate 4-vector, $x^\mu = (t, r_x, r_y, r_z)$,

  k^μ: momentum 4-vector, $k^\mu = (E, k_x, k_y, k_z)$, $k^\mu k_\mu = m^2$,

- **additional fields in relativistic hydro:**

  u^μ : velocity 4-vector, $u^\mu = \gamma (1, v_x, v_y, v_z)$, $u^\mu u_\mu = 1$,

  $g^{\mu\nu}$: metric tensor, $g^{\mu\nu}$= diag(1,-1,-1,-1),

  $T^{\mu\nu}$: energy-momentum tensor .

Nonrelativistic perfect fluid dynamics

- **Equations of nonrelativistic hydro:**
  - **local conservation of**

    charge: continuity

    momentum: Euler

    energy

$$\partial_t n + \nabla(n\mathbf{v}) = 0,$$
$$mn\left[\partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v}\right] = 0,$$
$$\partial_t \epsilon + \nabla(\epsilon \mathbf{v}) + p\nabla\mathbf{v} = 0.$$

- **EoS needed:**

$$p = nT\,, \qquad \epsilon = \kappa(T) nT,$$

- **Perfect fluid: 2 equivalent definitions, term used by PDG**

  # 1: no bulk and shear viscosities, and no heat conduction.

  # 2: $T^{\mu\nu}$ = diag(e,-p,-p,-p) in the local rest frame.

- **ideal fluid: ambiguously defined term, discouraged**

  #1: keeps its volume, but conforms to the outline of its container

  #2: an inviscid fluid

Dissipative, non-relativistic fluid dynamics

Navier-Stokes equations: dissipative, nonrelativistic hydro:

$$\begin{aligned}
\partial_t n + \nabla(n\mathbf{v}) &= 0, \\
mn\left[\partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v}\right] &= -\nabla p + \eta\left[\Delta\mathbf{v} + \frac{1}{3}\nabla(\nabla\mathbf{v})\right] + \zeta\nabla(\nabla\mathbf{v}), \\
\partial_t \epsilon + \nabla(\epsilon\mathbf{v}) + p\nabla\mathbf{v} &= \nabla(\lambda\nabla T) + \zeta(\nabla\mathbf{v})^2 + 2\eta\left[TrD^2 - \frac{1}{3}(\nabla\mathbf{v})^2\right],
\end{aligned}$$

$$D_{ik} = \frac{1}{2}\left(\frac{\partial v_i}{\partial r_k} + \frac{\partial v_k}{\partial r_i}\right).$$

EoS needed:

$$\begin{aligned}
p &= nT, \\
\epsilon &= \frac{1}{c_s^2(T)}p \equiv \kappa p,
\end{aligned}$$

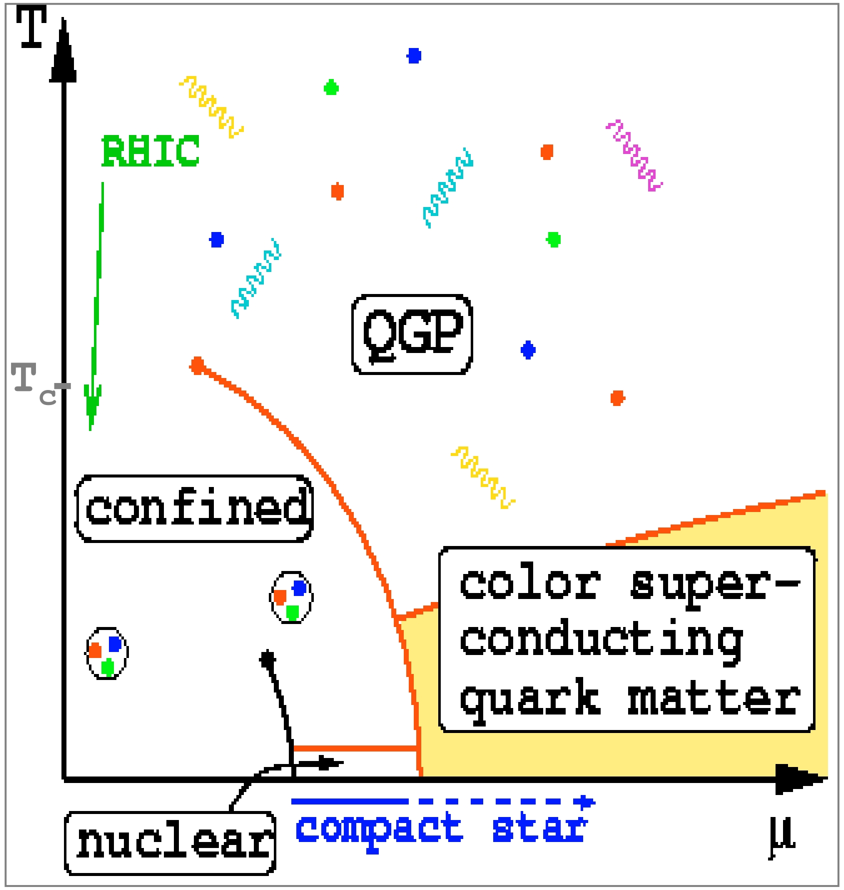
Shear and bulk viscosity, heat conduction effects:

$\boxed{\eta_S}$ $\boxed{\zeta}$ $\boxed{\lambda}$

Input from lattice: EoS of QCD Matter

Old idea: Quark Gluon Plasma
More recent: Liquid of quarks

T_c=176±3 MeV (~2 terakelvin)
(hep-ph/0511166)
at μ = 0, a cross-over
Aoki, Endrődi, Fodor, Katz, Szabó
hep-lat/0611014



LQCD input for hydro: $p(\mu,T)$
LQCD for RHIC region: $p \sim p(T)$,
$$c_s^2 = \delta p/\delta e = c_s^2(T) = 1/\kappa(T)$$
It's in the family exact hydro solutions!

New exact, parametric hydro solutions

Ansatz: the density n (and T and ε) depend on coordinates only through a scale parameter s

- T. Cs. Acta Phys. Polonica B37 (2006), hep-ph/0111139

$$n = f(t)g(s).$$

$$\begin{aligned} \partial_t n &= f'(t)g(s) + f(t)g'(s)\partial_t s, \\ \nabla(vn) &= f(t)g(s)\nabla v + f(t)g'(s)v\nabla s. \end{aligned}$$

Principal axis of ellipsoid:
(X,Y,Z) = (X(t), Y(t), Z(t))

$$\frac{f'(t)}{f(t)} = -\nabla v,$$

$$\partial_t s + v\nabla s = 0$$

$$f(t) = \frac{X_0 Y_0 Z_0}{XYZ}$$

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

$$v = \left(\frac{\dot X}{X}\, r_x, \frac{\dot Y}{Y}\, r_y, \frac{\dot Z}{Z}\, r_z \right)$$

Density=const on ellipsoids.
g(s): arbitrary scaling function.

Directional Hubble flow.
Notation: n ~ ν(s), T ~ τ(s) etc.

Perfect, ellipsoidal hydro solutions

A new family of PARAMETRIC, exact, scale-invariant solutions

T. Cs. Acta Phys. Polonica B37 (2006) hep-ph/0111139

Volume is introduced as V = XYZ

$$
\begin{aligned}
n(t,\mathbf{r}) &= n_0 \frac{V_0}{V} \nu(s) \\
\mathbf{v}(t,\mathbf{r}) &= \left(\frac{\dot X}{X} r_x, \frac{\dot Y}{Y} r_y, \frac{\dot Z}{Z} r_z \right) \\
T(t,\mathbf{r}) &= T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \mathcal{T}(s) \\
\nu(s) &= \frac{1}{\mathcal{T}(s)} \exp\left(- \frac{T_i}{2T_0} \int_0^s \frac{du}{\mathcal{T}(u)} \right)
\end{aligned}
$$

$$
s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}
$$

For κ = κ(T) exact solutions, see
T. Cs, S.V. Akkelin, Y. Hama,
B. Lukács, Yu. Sinyukov,
hep-ph/0108067, Phys.Rev.C67:034904,2003
or see the solutions of Navier-Stokes later on.

The dynamics is reduced to coupled, nonlinear but ordinary differential equations for the scales X,Y,Z

$$
X\ddot X = Y \ddot Y = Z \ddot Z = \frac{T_i}{m} \left(\frac{V_0}{V} \right)^{1/\kappa}
$$

Many hydro problems (initial conditions, role of EoS, freeze-out conditions)
can be easily **illustrated and understood** on the equivalent problem:
a classical potential motion of a mass-point in a conservative potential (a shot)!

Note: temperature scaling function τ(s) remains arbitrary! ν(s) depends on τ(s). -> FAMILY of solutions.

From fluid expansion to potential motion

Dynamics of pricipal axis:

The role of initial boundary conditions, EoS and freeze-out in hydro can be understood from potential motion!

Initial boundary conditions

From the new family of exact solutions, the initial conditions:

Initial coordinates:

(nuclear geometry + time of thermalization)

$$(X_0\, Y_0\, Z_0)$$

Initial velocities:

$$(\dot{X}_0\, \dot{Y}_0\, \dot{Z}_0)$$

(pre-equilibrium+ time of thermalization)

Initial temperature:

$$T_0$$

Initial density:

$$n_0$$

Initial profile function:

$$\tau(s)$$

(energy deposition and pre-equilibrium process)

Role of initial temperature profile

- **Initial temperature profile = arbitrary positive function**
- **Infinitly rich class of solutions**
- **Matching initial conditions for the density profile**
  - T. Cs. Acta Phys. Polonica B37 (2006) 1001, hep-ph/0111139

$$\nu(s) = \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0}\int_0^s \frac{du}{\mathcal{T}(u)}\right)$$

- **Homogeneous temperature $\Rightarrow$ Gaussian density**

$$\nu(s) = \exp(-s/2), \quad \mathcal{T}(s) = 1.$$

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

- **Buda-Lund profile:**

$$\mathcal{T}(s) = \frac{1}{1+bs}$$

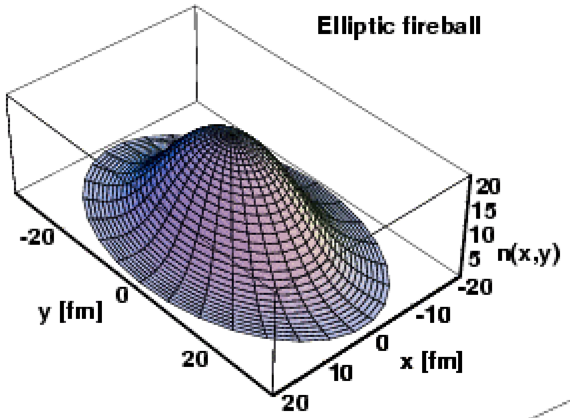
$$\nu(s) = (1+bs)\exp\left[-\frac{T_i}{2T_0}(s + bs^2/2)\right]$$

Zimányi-Bondorf-Garpman profile:

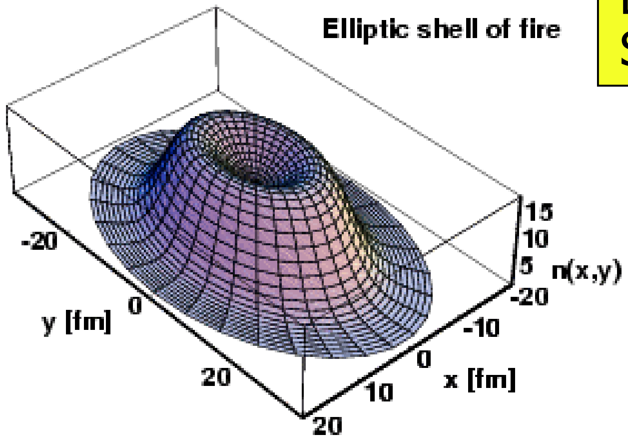
$$\mathcal{T}(s) = (1-s)\,\Theta(1-s)$$

$$\nu(s) = (1-s)^{\alpha}\,\Theta(1-s)$$

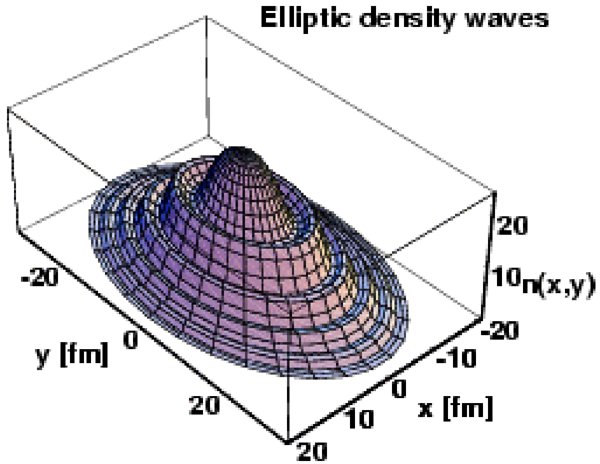
Illustrated initial T-> density profiles



Determines density profile!
Examples of density profiles
- Fireball
- Ring of fire
- Embedded shells of fire
Exact integrals of hydro
Scales expand in time

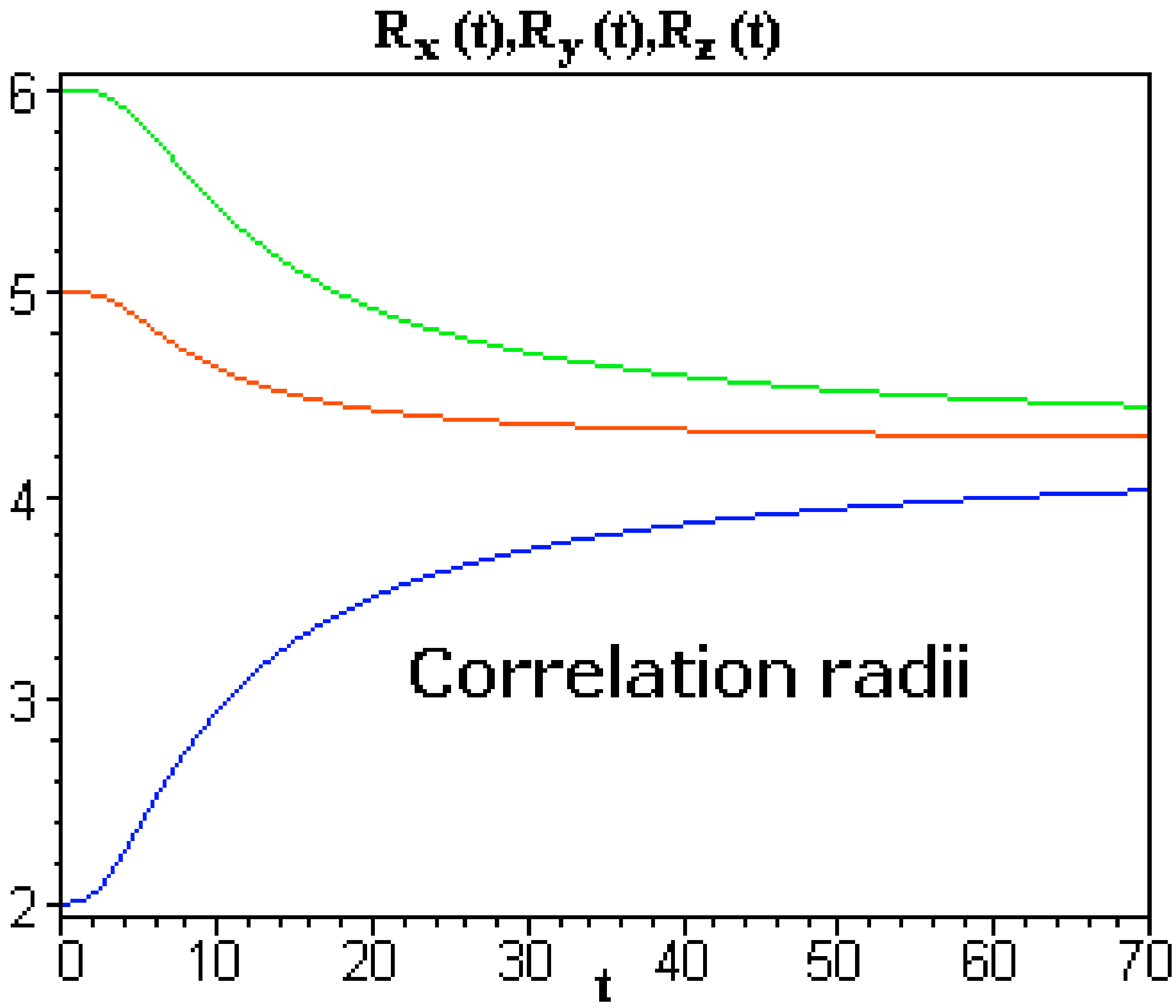


Time evolution of the scales (X,Y,Z)
follows a classic potential motion.
Scales at freeze out -> observables.
info on history LOST!
No go theorem - constraints
on initial conditions
(penetrating probels) indispensable.



Illustrations of exact hydro results

- **Propagate the hydro solution in time numerically:**



Final (freeze-out) boundary conditions

From the new exact hydro solutions, the conditions to stop the evolution:

Freeze-out temperature: T_f

Final coordinates: $(X_f \, Y_f \, Z_f)$

(cancel from measurables, diverge)

Final velocities: $(\dot{X}_f \, \dot{Y}_f \, \dot{Z}_f)$

(determine observables, tend to constants)

Final density: n_f

(cancels from measurables, tends to 0)

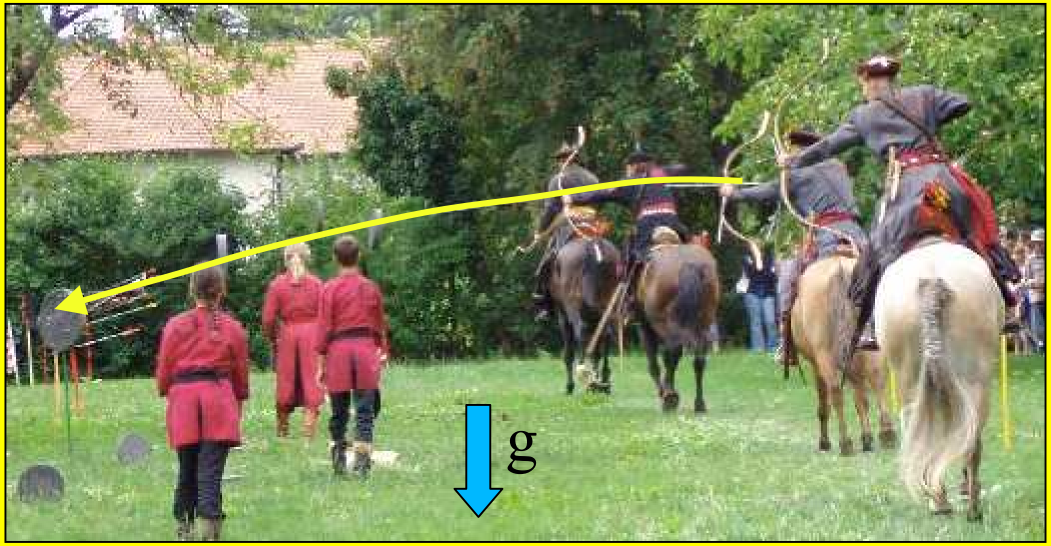
Final profile function: $\tau(s)$

(= initial profile function! from solution)

Role of the Equation of States:

The potential depends
on $\kappa = \delta\varepsilon / \delta p$**:**

$$T_0 \left(\frac{V_0}{V} \right)^{1/\kappa}$$



Time evolution of the scales (X,Y,Z) follows a classic potential motion.
Scales at freeze out determine the observables. Info on history LOST!
No go theorem - constraints on initial conditions
(information on spectra, elliptic flow of penetrating probels) indispensable.

The arrow hits the target, but can one determine g from this information??

Initial conditions <-> Freeze-out conditions:

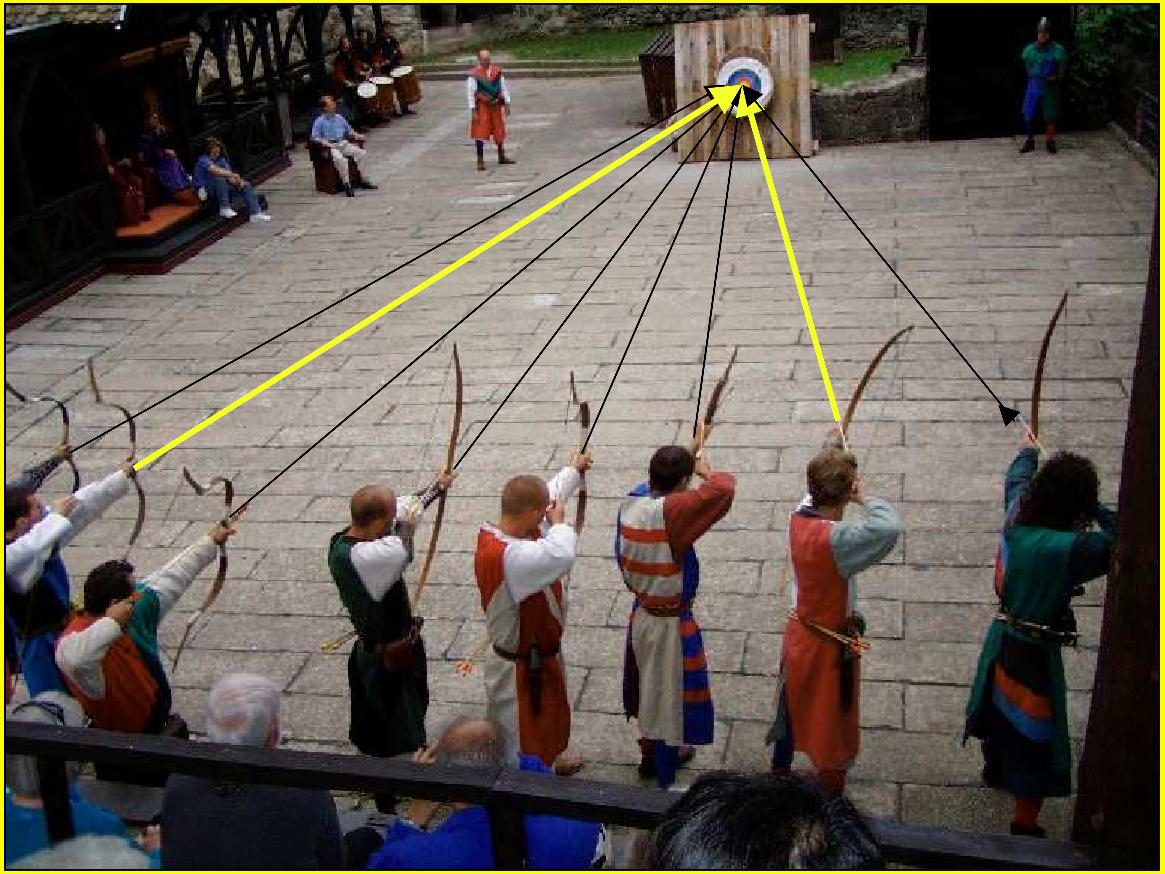
Different initial conditions

but

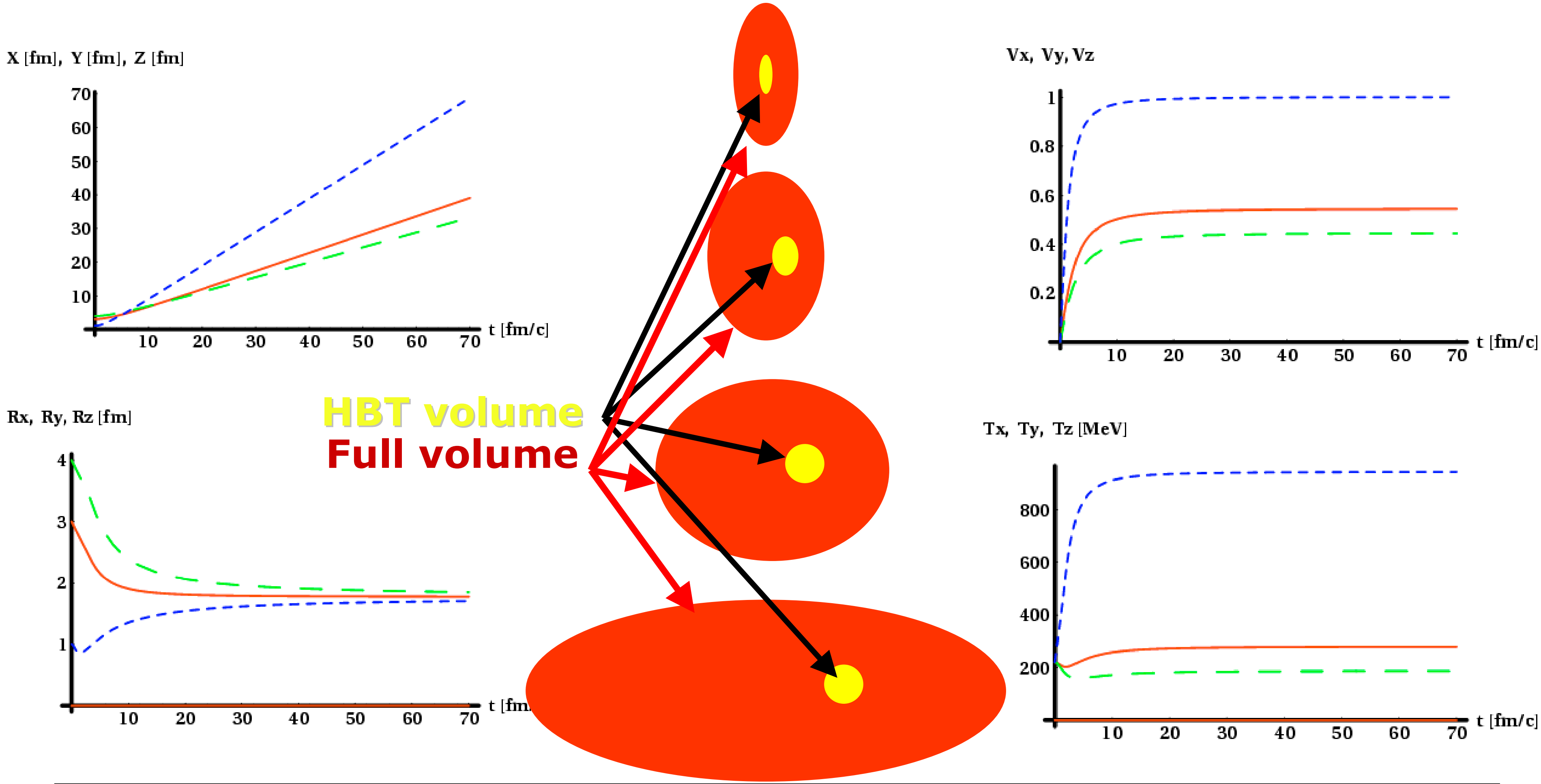
same freeze-out conditions

ambiguity!

Penetrating probes radiate through the time evolution!



Solution of the "HBT puzzle"



Geometrical sizes keep on increasing. Expansion velocities tend to constants.
HBT radii R_x, R_y, R_z approach a direction independent constant.
Slope parameters tend to direction dependent constants.
General property, independent of initial conditions - a beautiful exact result.

Understanding hydro results

New exact solutions of 3d nonrelativistic hydrodynamics:

Hydro problem equivalent to potential motion (a shot)!

Hydro:		**Shot of an arrow:**
Desription of data	⟷	Hitting the target
Initial c		velocity
Equation		ntial
Freeze-o		t
Data co		lls
		tial (?)
Differen		n
exactly t		aneously (!)
EoS and		n be
		ne potential
Universal scaling of v_2	⟷	**In a perfect shot, the shape of trajectory is a parabola**
Viscosity effects	⟷	Drag force of air
numerical hydro disagrees with data	⟷	Arrow misses the target (!), see C. Ogilvie's talk

Dissipative, ellipsoidal hydro solutions

A new family of dissipative, exact, scale-invariant solutions

T. Cs. and Y. Hama, in preparation ...

Volume is V = XYZ

$$n(t,\mathbf{r}) = n_0 \frac{V_0}{V} \nu(s)$$

$$\mathbf{v}(t,\mathbf{r}) = \left(\frac{\dot X}{X} r_x, \frac{\dot Y}{Y} r_y, \frac{\dot Z}{Z} r_z \right)$$

$$\boxed{T(t,\mathbf{r}) = T_0 f(t) \mathcal{T}(s),}$$

$$\nu(s) = \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0} \int_0^s \frac{du}{\mathcal{T}(u)} \right)$$

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

The dynamics is reduced to coupled, nonlinear but ordinary differential equations for the scales X,Y,Z

$$\boxed{X\ddot X = Y\ddot Y = Z\ddot Z = \frac{T_i f(t)}{m}}$$

$$\boxed{T_0 f(t) = T(t) \equiv T}$$

Even VISCOUS hydro problems (initial conditions, role of EoS, freeze-out conditions, DISSIPATION) can be easily illustrated and understood on the equivalent problem:

a classical potential motion of a mass-point in a conservative potential (a shot)!

Note: temperature scaling function $\tau(s)$ remains arbitrary! $\nu(s)$ depends on $\tau(s)$. -> FAMILY of solutions.

Dissipative, ellipsoidal hydro solutions

A new family of PARAMETRIC, exact, scale-invariant solutions

T. Cs. and Y. Hama, in preparation

Introduction of kinematic bulk and shear viscosity coefficients:

$$\nu_S = \frac{\eta}{mn} = c_1$$
$$\nu_B = \frac{\zeta}{mn} = c_2$$

Note that the Navier-Stokes (gen. Euler) is automatically solved by the directional Hubble ansatz, as the 2nd gradients of the velocity profile vanish!

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T_i f(t)}{m}$$

Only non-trivial contribution from the energy equation:

$$\dot{T} - \dot{T}\frac{d\ln c_s^2(T)}{d\ln T} = -c_s^2(T)T\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right) + m\nu_B\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)^2 + \\ + 2m\nu_S\left[\left(\frac{\dot{X}}{X}\right)^2+\left(\frac{\dot{Y}}{Y}\right)^2+\left(\frac{\dot{Z}}{Z}\right)^2-\frac{1}{3}\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)^2\right]$$

Asymptotics: T -> 0 for large times, hence X ~ t, Y ~ t, Z ~ t, and asymptotic analysis possible!

EOS: drives dynamics, asymptotically dominant term: perfect fluid!!

Shear: asymptotically sub-subleading correction, ~ $1/t^3$

bulk: asymptotically sub-leading correction, ~ $1/t^2$

Dissipative, heat conductive hydro solutions

A new family of PARAMETRIC, exact, scale-invariant solutions

T. Cs. and Y. Hama, in preparation

Introduction of 'kinematic' heat conductivity:

$$\nu_Q = \frac{\lambda}{mn} = c_3$$

The Navier-Stokes (gen. Euler) is again automatically solved by the directional Hubble ansatz!

Only non-trivial contribution from the energy equation:

$$\begin{aligned}\dot{T} - \dot{T}\frac{d\ln c_s^2(T)}{d\ln T} \approx\ & -c_s^2(T)T\left(\frac{\dot X}{X}+\frac{\dot Y}{Y}+\frac{\dot Z}{Z}\right) + m\nu_B\left(\frac{\dot X}{X}+\frac{\dot Y}{Y}+\frac{\dot Z}{Z}\right)^2 + \\ & + 2m\nu_S\left[\left(\frac{\dot X}{X}\right)^2+\left(\frac{\dot Y}{Y}\right)^2+\left(\frac{\dot Z}{Z}\right)^2-\frac{1}{3}\left(\frac{\dot X}{X}+\frac{\dot Y}{Y}+\frac{\dot Z}{Z}\right)^2\right] + \\ & + m\left[\nu_Q T_i T'(0)\left(\frac{1}{X^2}+\frac{1}{Y^2}+\frac{1}{Z^2}\right)\right]\end{aligned}$$

$$\nabla\nu(s) = 0$$

$$\Delta T \approx -T_i\left(\frac{1}{X^2}+\frac{1}{Y^2}+\frac{1}{Z^2}\right)$$

Role of heat conduction can be followed asymptotically

- same order of magnitude ($1/t^2$) as bulk viscosity effects
- valid only for nearly constant densities,
- destroys self-similarity of the solution if there are strong irregularities in temperature

Scaling predictions, for (viscous) fluid dynamics

$$T'_x = T_f + m\dot{X}_f^2 \ ,$$
$$T'_y = T_f + m\dot{Y}_f^2 \ ,$$
$$T'_z = T_f + m\dot{Z}_f^2 \ .$$

- Slope parameters increase linearly with mass
- Elliptic flow is a universal function its variable w is proportional to transverse kinetic energy and depends on slope differences.

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

$$w = \frac{k_t^2}{4m}\left(\frac{1}{T'_y} - \frac{1}{T_x}\right),$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

Inverse of the HBT radii increase linearly with mass analysis shows that they are asymptotically the same

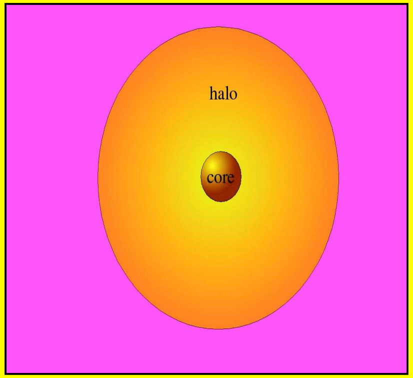
Relativistic correction: m -> m_t

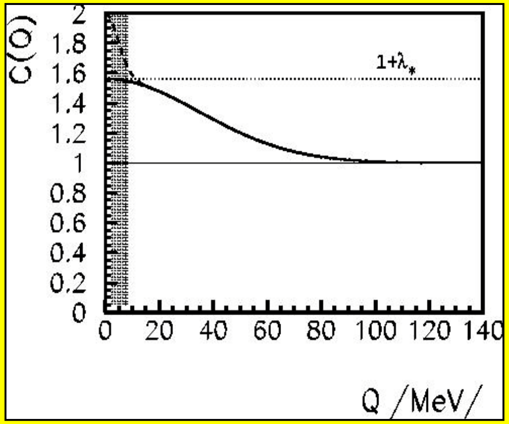
hep-ph/0108067,
nucl-th/0206051

$$R'^{-2}_x = X_f^{-2}\left(1 + \frac{m}{T_f}\dot{X}_f^2\right),$$
$$R'^{-2}_y = Y_f^{-2}\left(1 + \frac{m}{T_f}\dot{Y}_f^2\right),$$
$$R'^{-2}_z = Z_f^{-2}\left(1 + \frac{m}{T_f}\dot{Z}_f^2\right).$$

Principles for Buda-Lund hydro model

- **Analytic expressions for all the observables**
- **3d expansion, local thermal equilibrium, symmetry**
- **Goes back to known exact hydro solutions:**
  - **nonrel, Bjorken, and Hubble limits, 1+3 d ellipsoids**
  - **but phenomenology, extrapolation for unsolved cases**
- **Separation of the Core and the Halo**
  - **Core: perfect fluid dynamical evolution**
  - **Halo: decay products of long-lived resonances**
- **Missing links: phenomenology needed**
  - **search for accelerating ellipsoidal rel. solutions**
  - **first accelerating rel. solution: nucl-th/0605070**

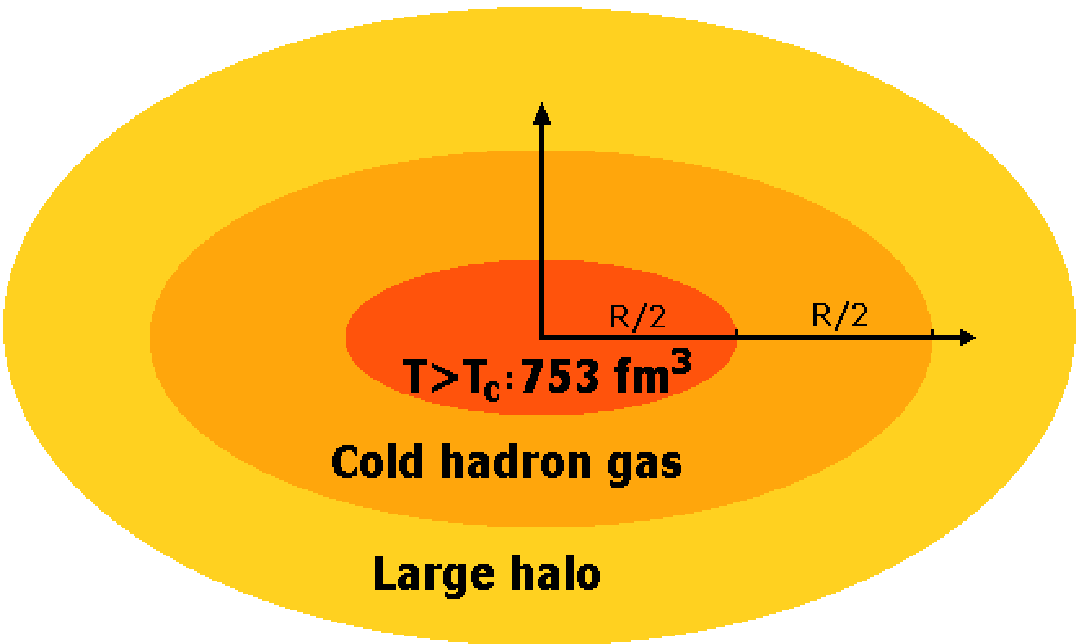


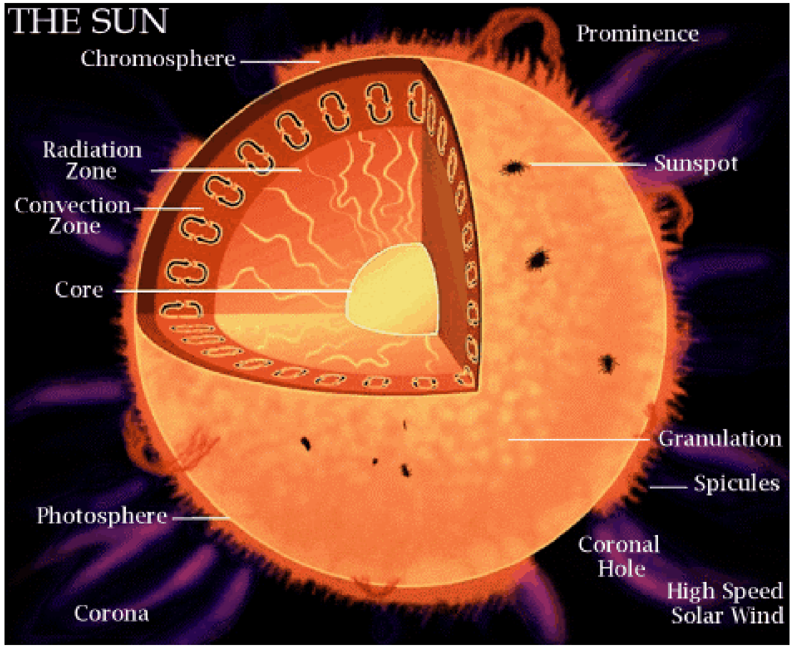


A useful analogy

Fireball at RHIC ⇔ our Sun

- Core ⇔ Sun
- Halo ⇔ Solar wind
- $T_{0,RHIC}$ ~ 210 MeV ⇔ $T_{0,SUN}$ ~ 16 million K
- $T_{surface,RHIC}$ ~ 100 MeV ⇔ $T_{surface,SUN}$ ~6000 K





Buda-Lund hydro model

The general form of the emission function:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

Calculation of observables with core-halo correction:

$$\mathrm{N}_1(p) = \frac{1}{\sqrt{\lambda_*}} \int d^4x S_c(p,x)$$

$$C(Q,p) = 1 + \left|\frac{\tilde{S}(Q,p)}{\tilde{S}(0,p)}\right|^2 = 1 + \lambda_* \left|\frac{\tilde{S}_c(Q,p)}{\tilde{S}_c(0,p)}\right|^2$$

Assuming profiles for

flux, temperature, chemical potential and flow

Buda-Lund model is ***fluid dynamical***

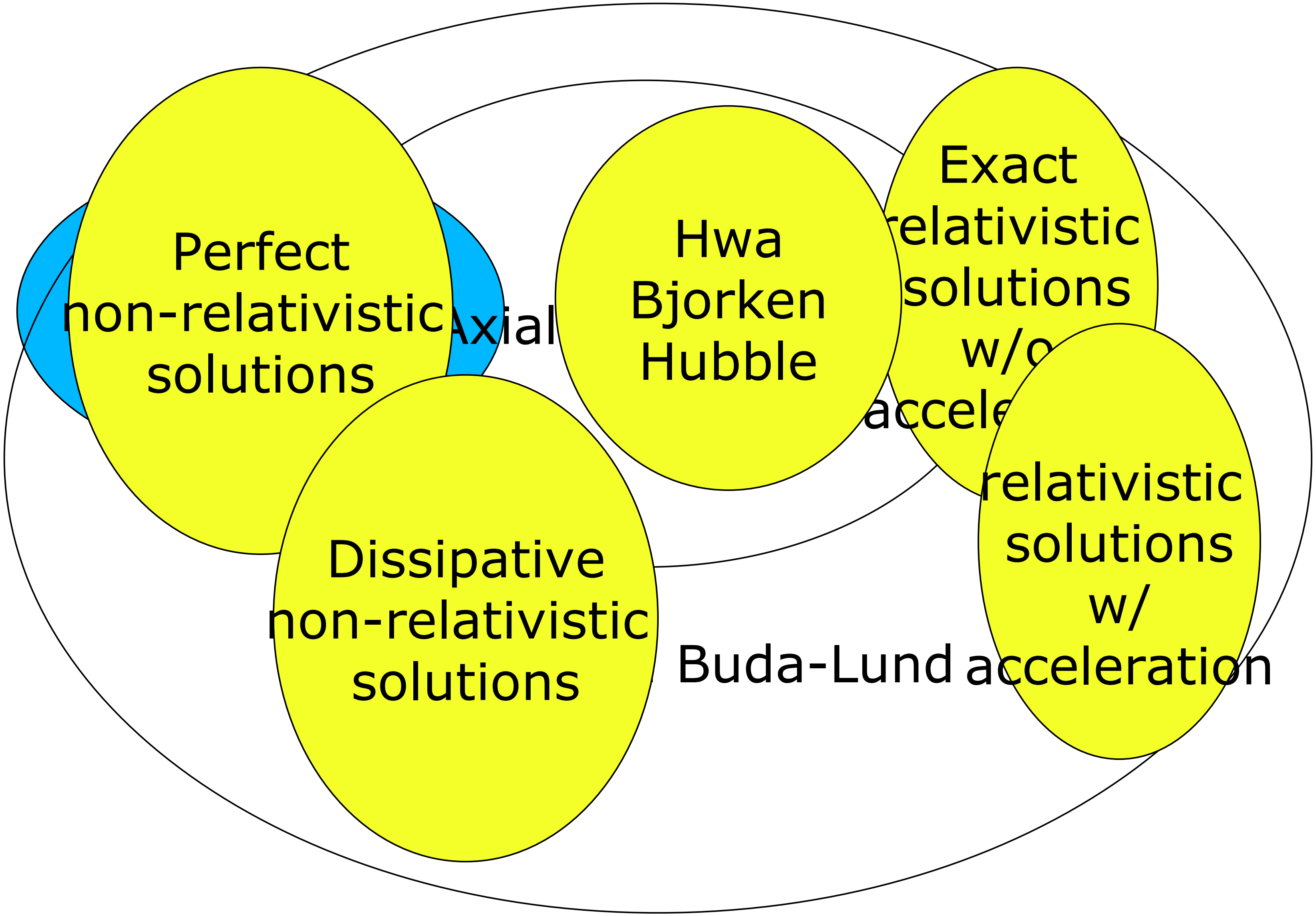
First formulation: parameterization
based on the flow profiles of
- Zimanyi-Bondorf-Garpman non-rel. exact sol.
- Bjorken rel. exact sol.
- Hubble rel. exact sol.

Remarkably successfull in describing
h+p and A+A collisions at CERN SPS and at RHIC

led to the discovery of an incredibly rich family of
parametric, *exact solutions* of
- non-relativistic, perfect hydrodynamics
- imperfect hydro with bulk + shear viscosity + heat conductivity
- relativistic hydrodynamics, finite $dn/d\eta$ and initial acceleration
- all cases: with temperature profile !

Further research: relativistic ellipsoidal exact solutions
with acceleration and dissipative terms

Buda-Lund and exact hydro sols



Scaling predictions: Buda-Lund hydro

$$T_x = T_0 + \overline{m}_t \dot{X}^2 \frac{T_0}{T_0 + \overline{m}_t a^2},$$

$$\overline{m}_t = m_t \cosh(\eta_s - y).$$

- Slope parameters increase linearly with transverse mass
- Elliptic flow is same universal function.
- Scaling variable w is prop. to generalized transv. kinetic energy and depends on effective slope diffs.

$$\varepsilon = \frac{T_x - T_y}{T_x + T_y}.$$

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

$$E_K = \frac{p_t^2}{2\overline{m}_t}$$

$$\frac{1}{T_*} = \frac{1}{2}\left(\frac{1}{T_x} + \frac{1}{T_y}\right).$$

Inverse of the HBT radii increase linearly with mass analysis shows that they are asymptotically the same

Relativistic correction: m -> m_t

hep-ph/0108067, nucl-th/0206051

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s,p)}{B(x_s,p)+s_q}\left(\frac{1}{X_i^2} + \frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2} + \frac{\dot{X}_i^2}{X_i^2}\right)$$

Hydro scaling of slope parameters

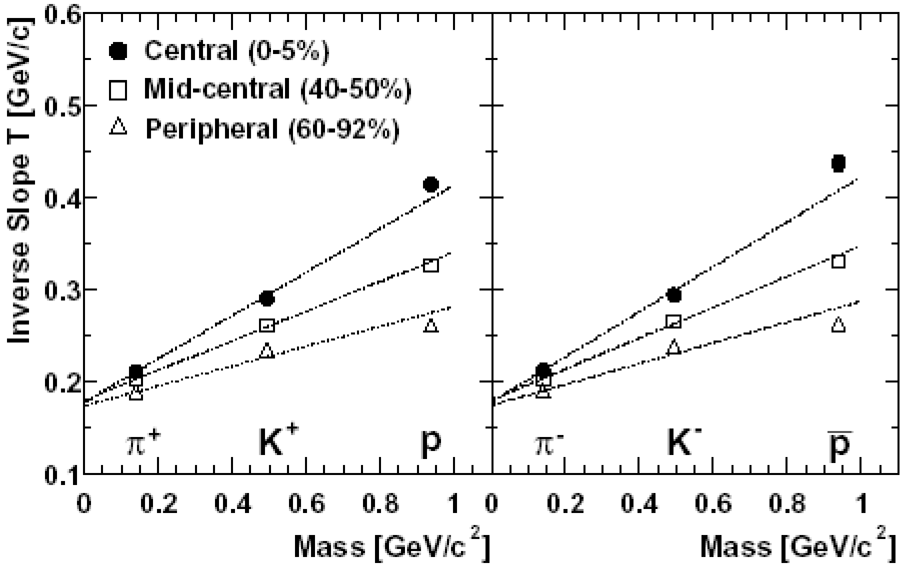
Buda-Lund hydro prediction:

$$T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

Exact non-rel. hydro solution:

$$T'_x = T_f + m\dot{X}_f^2 \, ,$$
$$T'_y = T_f + m\dot{Y}_f^2 \, ,$$
$$T'_z = T_f + m\dot{Z}_f^2 \, .$$

PHENIX data:



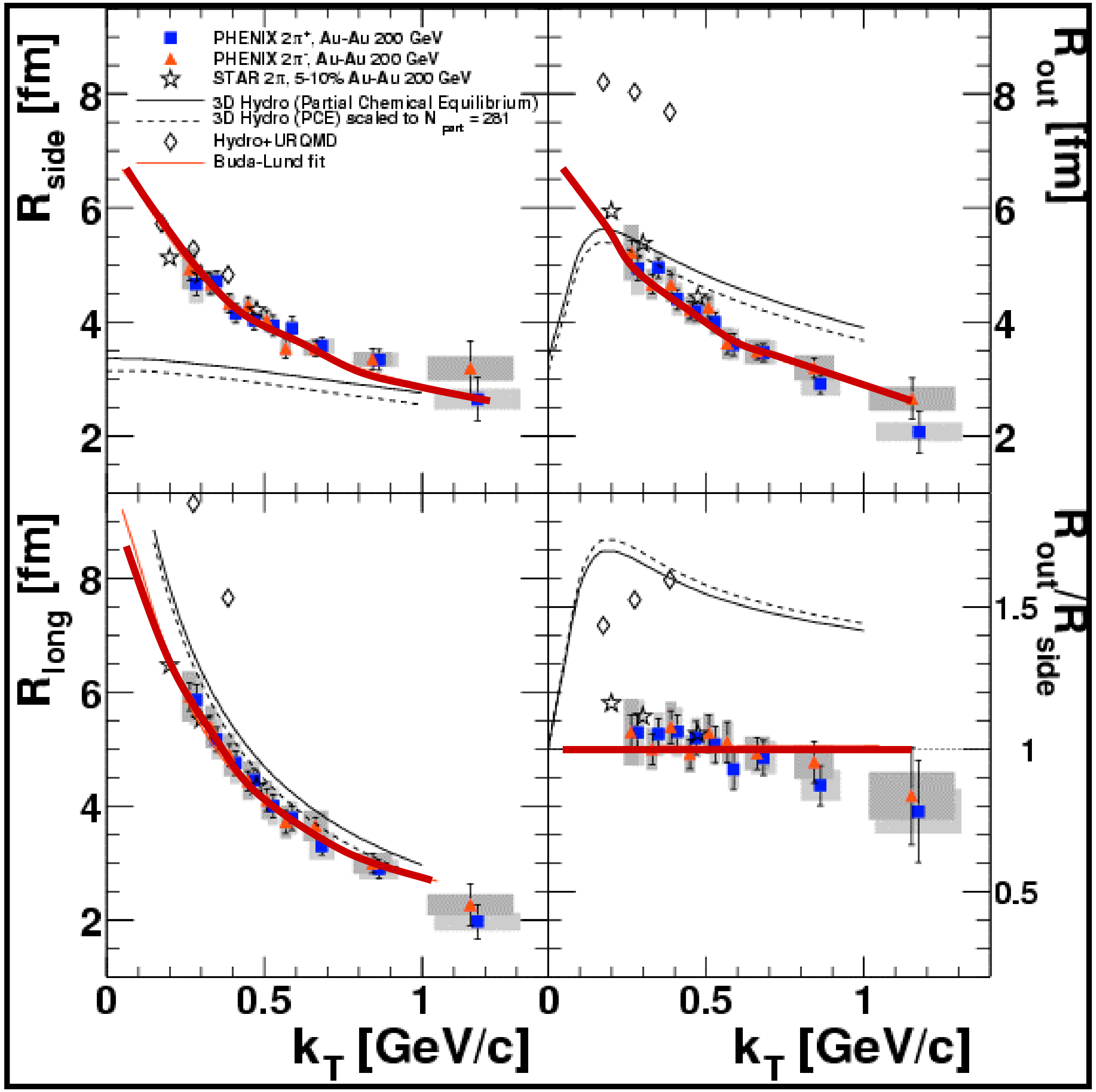
Femptoscopy signal of sudden hadronization

Buda-Lund hydro
fit indicates
hydro predicted
(1994-96)
scaling of HBT radii

T. Cs, L.P. Csernai
hep-ph/9406365
T. Cs, B. Lörstad
hep-ph/9509213

Hadrons with $T>T_c$:
a hint for
cross-over

M. Csanád, T. Cs, B. Lörstad and A. Ster,
nucl-th/0403074



The generalized Buda-Lund model

The original model was for axial symmetry only, central coll.

In its general hydrodynamical form:

Based on 3d relativistic and non-rel solutions of perfect fluid dynamics:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3}\frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

Have to assume special shapes:

Generalized Cooper-Frye prefactor:

$$p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x) H(\tau) d^4x$$

$$H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}} \exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

Four-velocity distribution:

$$u^\mu = (\gamma, \sinh\eta_x, \sinh\eta_y, \sinh\eta_z)$$

Temperature:

$$\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s} s\right)\left(1 + \frac{T_0 - T_e}{T_e}\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

Fugacity:

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$$

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

Some analytic Buda-Lund results

HBT radii widths:

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s,p)}{B(x_s,p)+s_q}\left(\frac{1}{X_i^2}+\frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2}+\frac{\dot{X}_i^2}{X_i^2}\right)$$

$$a^2 = \frac{T_0-T_s}{T_s} = \left\langle\frac{\Delta T}{T}\right\rangle_r$$

Slopes, effective temperatures:

$$\overline{m}_t = m_t\cosh(\eta_s - y).$$

$$\frac{1}{T_*} = \frac{1}{2}\left(\frac{1}{T_x}+\frac{1}{T_y}\right).$$

$$T_x = T_0 + \overline{m}_t\,\dot{X}^2\frac{T_0}{T_0+\overline{m}_t a^2},$$

Flow coefficients are <u>universal</u>:

$$\varepsilon = \frac{T_x-T_y}{T_x+T_y}.$$

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$

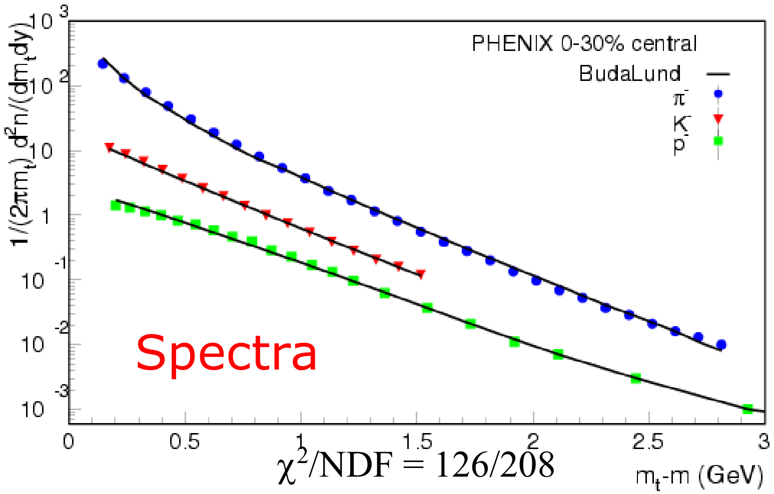
$$v_{2n+1} = 0$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

$$E_K = \frac{p_t^2}{2\overline{m}_t}$$

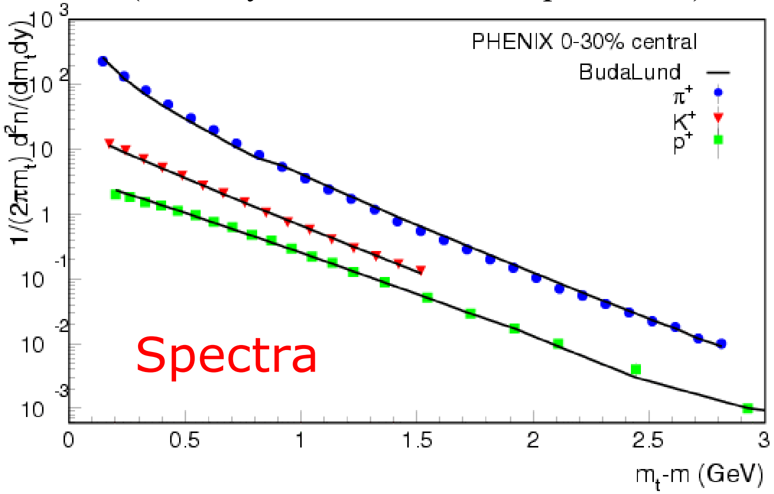
Buda-Lund hydro and Au+Au@RHIC

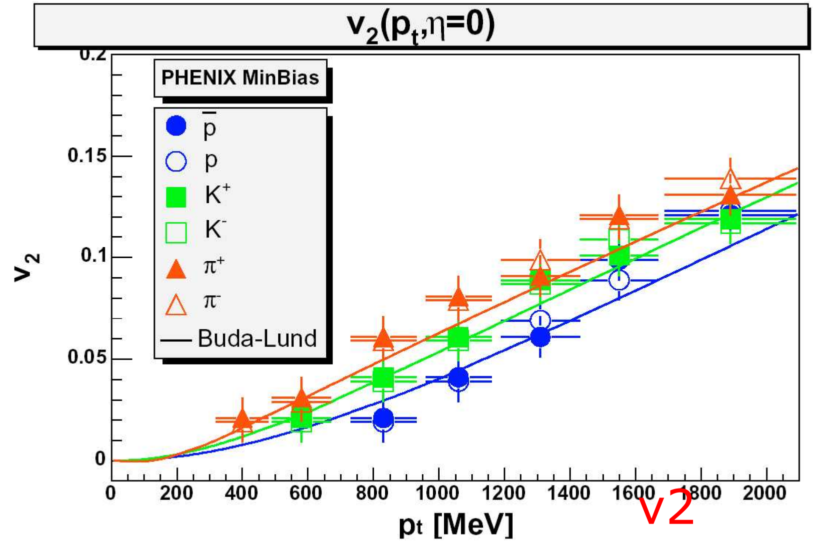
BudaLund v1.5 hydro fits to 200 AGeV Au+Au

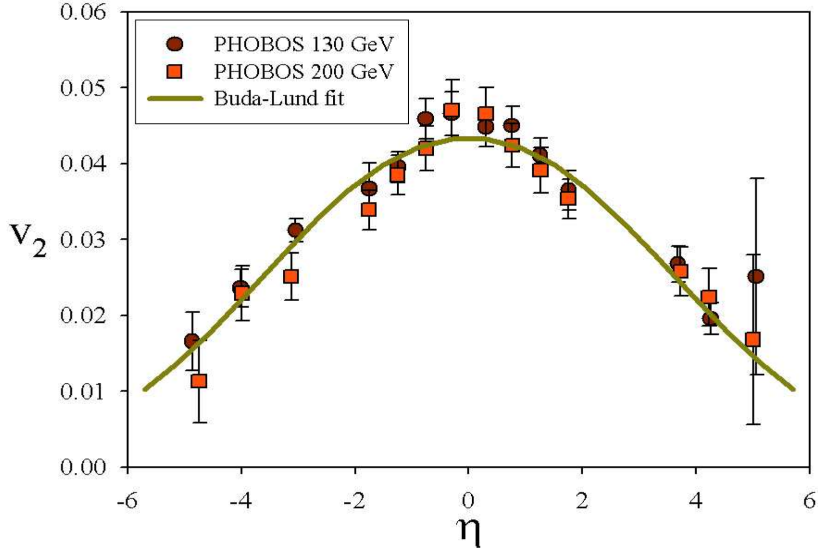


χ^2/NDF = 126/208

(stat + syst errors added in quadrature)

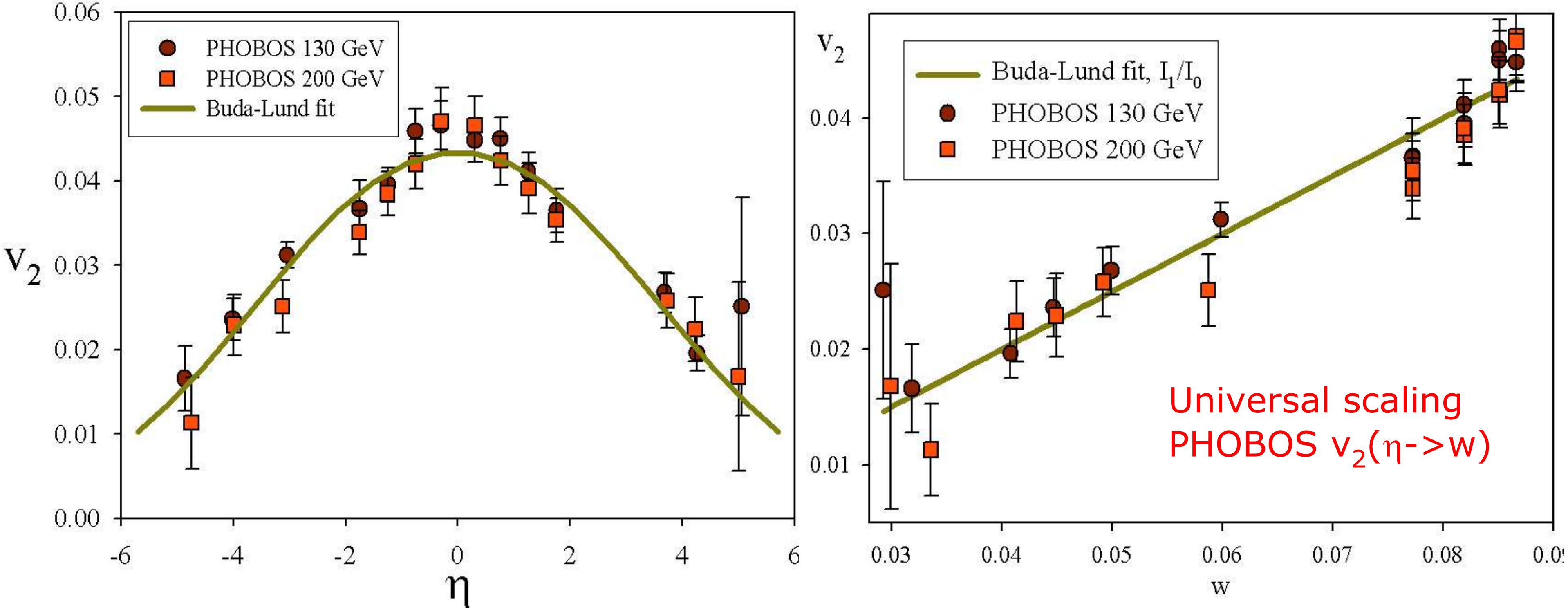






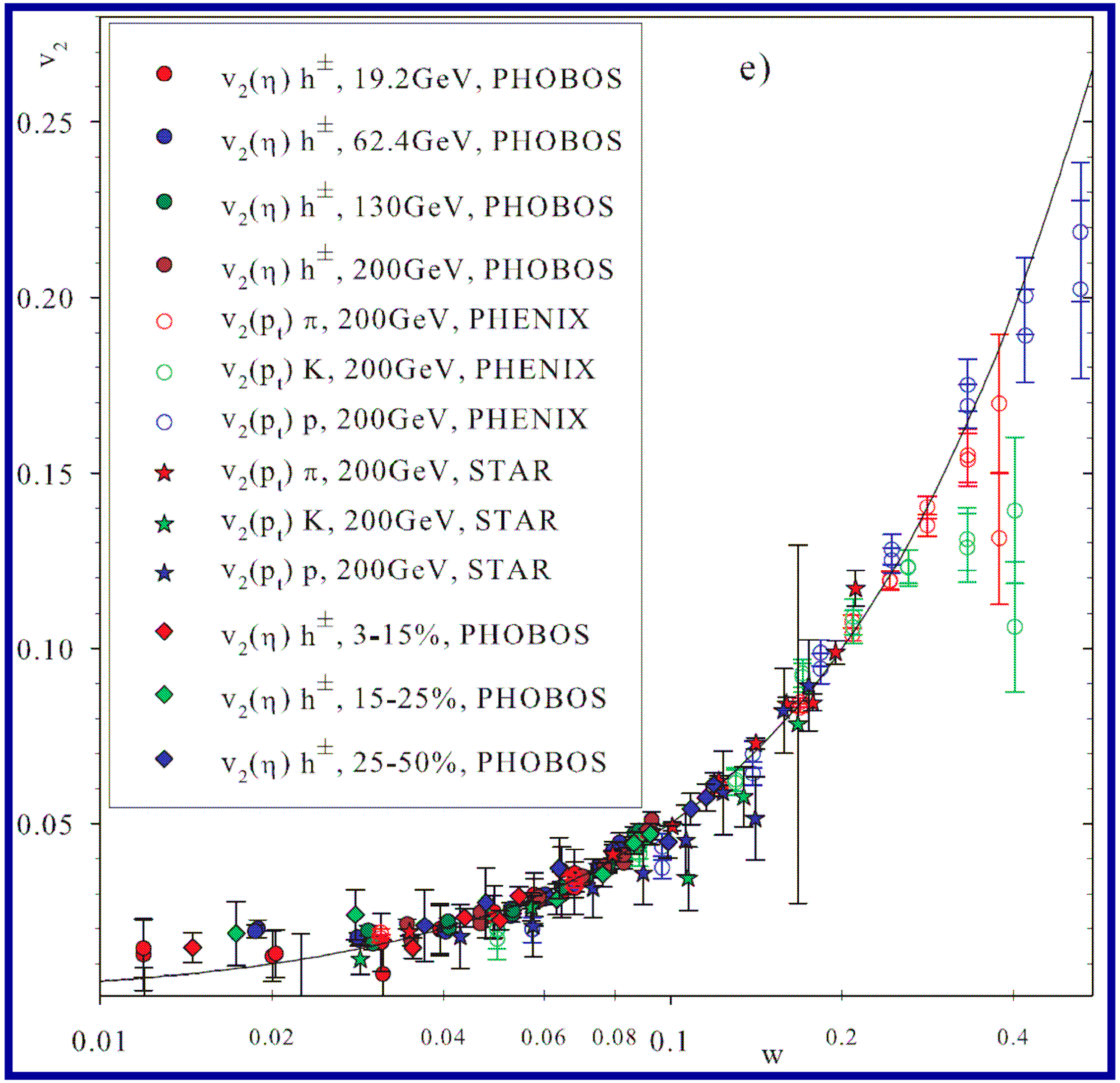
nucl-th/0311102, nucl-th/0207016, nucl-th/0403074

Confirmation



see nucl-th/0310040 and nucl-th/0403074,
R. Lacey@QM2005/ISMD 2005
A. Ster @ QM2005.

Universal hydro scaling of v_2

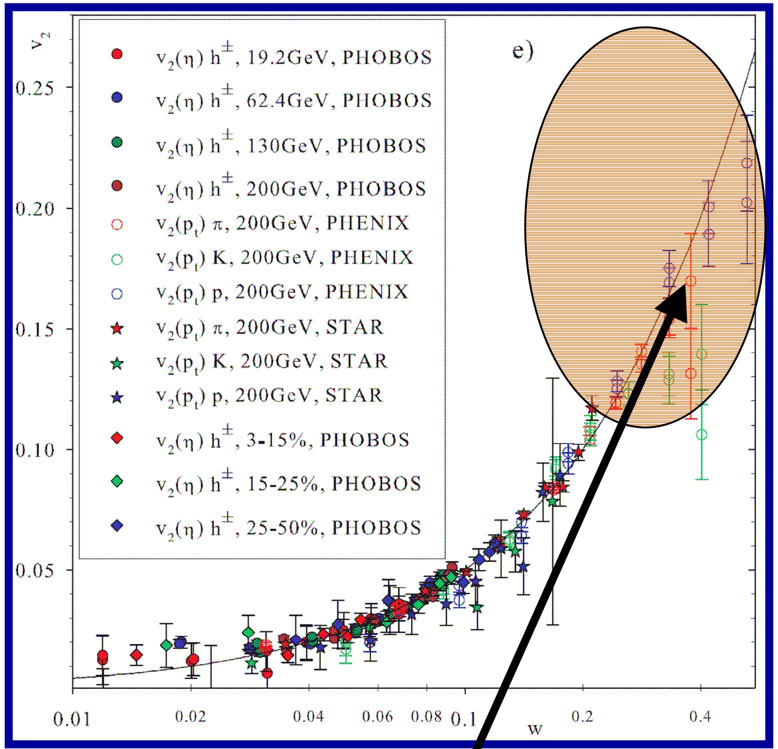


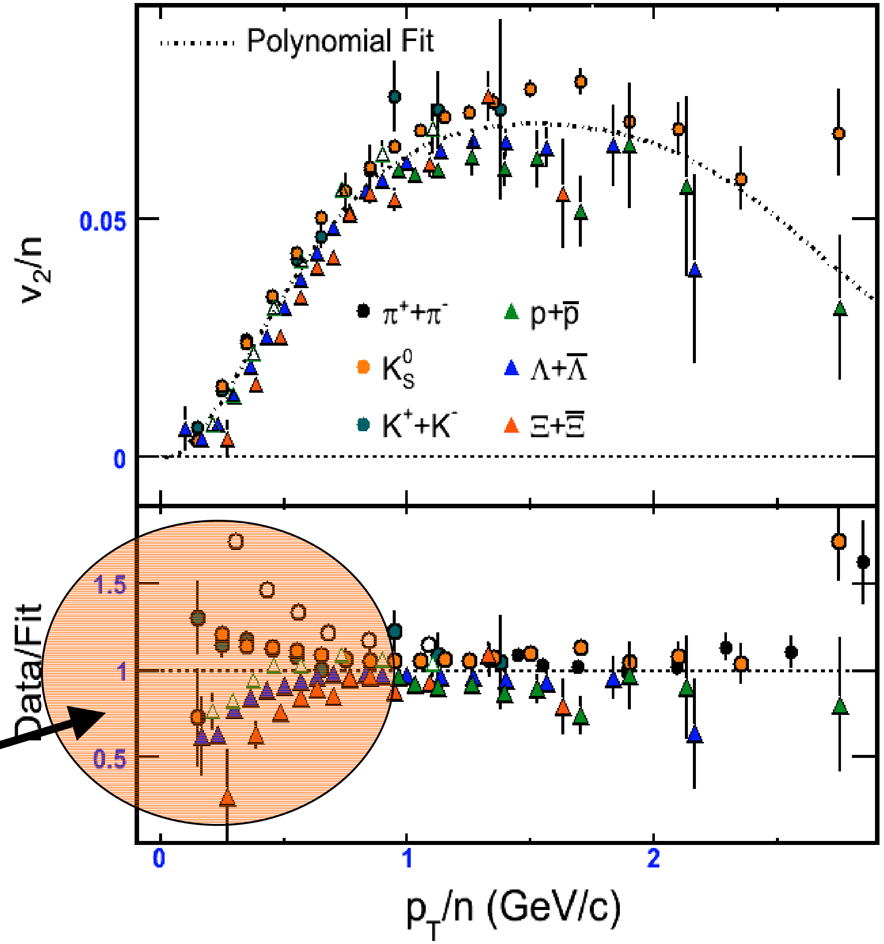
Black line: Theoretically predicted, universal scaling function from analytic works on perfect fluid hydrodynamics:

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

hep-ph/0108067, nucl-th/0310040

Scaling and scaling violations





Universal hydro scaling breaks where scaling with number of VALENCE QUARKS sets in, p_t ~ 1-2 GeV
Fluid of QUARKS!!

R. Lacey and M. Oldenburg, proc. QM'05
A. Taranenko et al,
PHENIX: nucl-ex/0608033

Summary

Au+Au elliptic flow data at RHIC satisfy the
UNIVERSAL scaling laws
predicted
(2001, 2003)
by the (Buda-Lund) hydro model,
based on exact solutions of
PERFECT FLUID hydrodynamics:
quantitative evidence for a perfect fluid in Au+Au at RHIC
scaling breaks, in p_t > 1.5 GeV, at ~|y| > y_{may} - 0.5

New, rich families of exact hydrodynamical solutions
discovered when searching for dynamics in Buda-Lund
- non-relativisitic perfect fluids
- non-relativistic, Navier-Stokes
- relativistic perfect fluids -> see M. Nagy's talk

Discovering New Laws

"In general we look for a new law by the following process.
First we guess it.
Then we compare the consequences of the guess to see
what would be implied if this law that we guessed is right.
Then we compare the result of the computation to nature,
with experiment or experience, compare it directly with observation,
to see if it works.
If it disagrees with experiment it is wrong.

In that simple statement is the key to science.
It does not make any difference how beautiful your guess is.
It does not make any difference how smart you are,
who made the guess, or what his name is —
if it disagrees with experiment it is wrong."

/R.P. Feynman/

Backup slides from now on

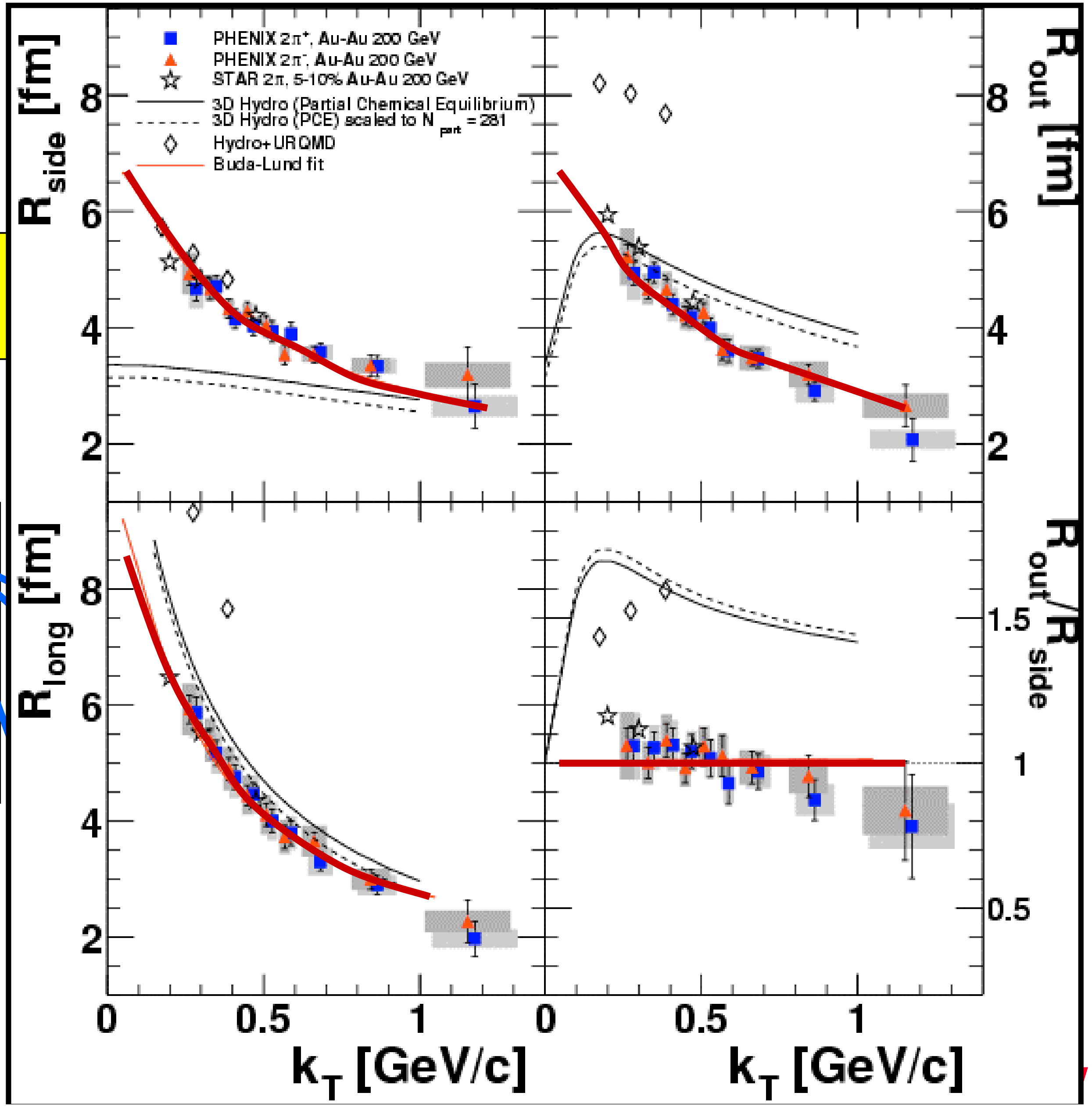
Hydro scaling of Bose-Einstein/HBT radii

$1/R^2_{eff}=1/R^2_{geom}+1/R^2_{thrm}$
and $1/R^2_{thrm} \sim m_t$

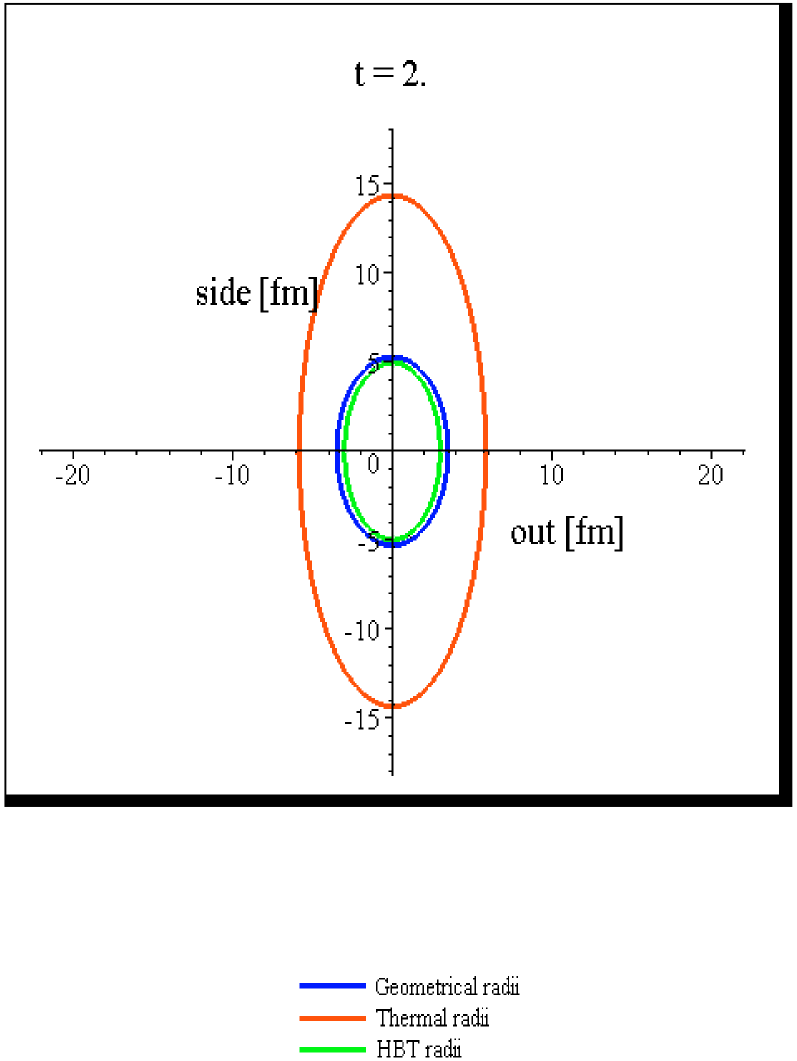
intercept is nearly 0, indicating $1/R_G^2 \sim 0$,

thus $\mu(x)/T(x)$ = const!

reason for success of thermal models @ RHIC!



Geometrical & thermal & HBT radii



3d analytic hydro: exact time evolution

geometrical size (fugacity ~ const)
Thermal sizes (velocity ~ const)
HBT sizes (phase-space density ~ const)

HBT dominated by the smaller of the geometrical and thermal scales

nucl-th/9408022, hep-ph/9409327
hep-ph/9509213, hep-ph/9503494

HBT radii approach a constant of time
HBT volume becomes spherical
HBT radii -> thermal ~ constant sizes

hep-ph/0108067, nucl-th/0206051
animation by Máté Csanád

Exact scaling laws of non-rel hydro

$$T'_x = T_f + m\dot{X}_f^2 \,,$$
$$T'_y = T_f + m\dot{Y}_f^2 \,,$$
$$T'_z = T_f + m\dot{Z}_f^2 \,.$$

- Slope parameters increase linearly with mass
- Elliptic flow is a universal function and variable w is proportional to transverse kinetic energy and depends on slope differences.

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

$$w = \frac{k_t^2}{4m}\left(\frac{1}{T'_y} - \frac{1}{T_x}\right),$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

Inverse of the HBT radii increase linearly with mass analysis shows that they are asymptotically the same

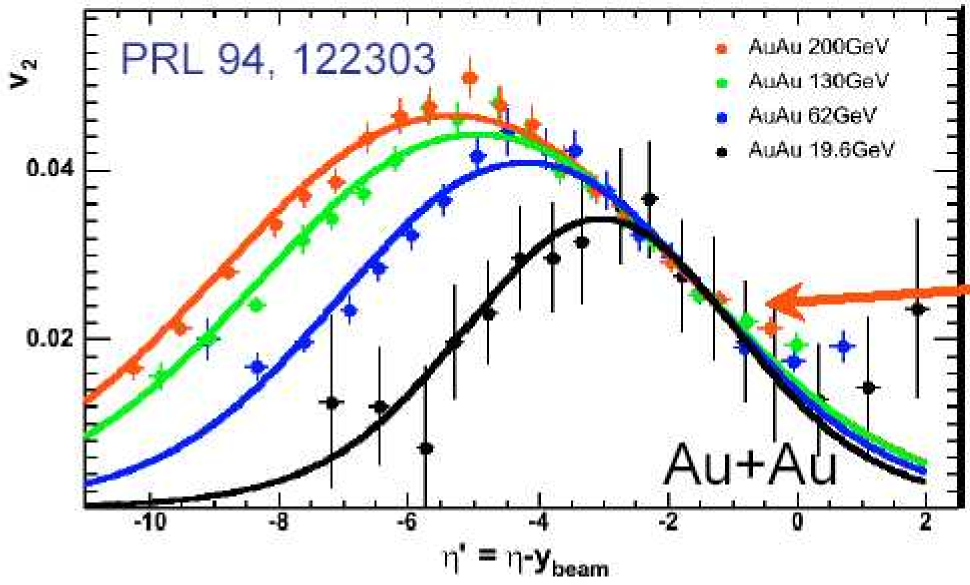
Relativistic correction: m -> m_t

hep-ph/0108067, nucl-th/0206051

$$R'^{-2}_x = X_f^{-2}\left(1 + \frac{m}{T_f}\dot{X}_f^2\right),$$
$$R'^{-2}_y = Y_f^{-2}\left(1 + \frac{m}{T_f}\dot{Y}_f^2\right),$$
$$R'^{-2}_z = Z_f^{-2}\left(1 + \frac{m}{T_f}\dot{Z}_f^2\right).$$

Hydro scaling of elliptic flow

Extended longitudinal scaling: v_2



A surprising scaling!

Not an initial state effect

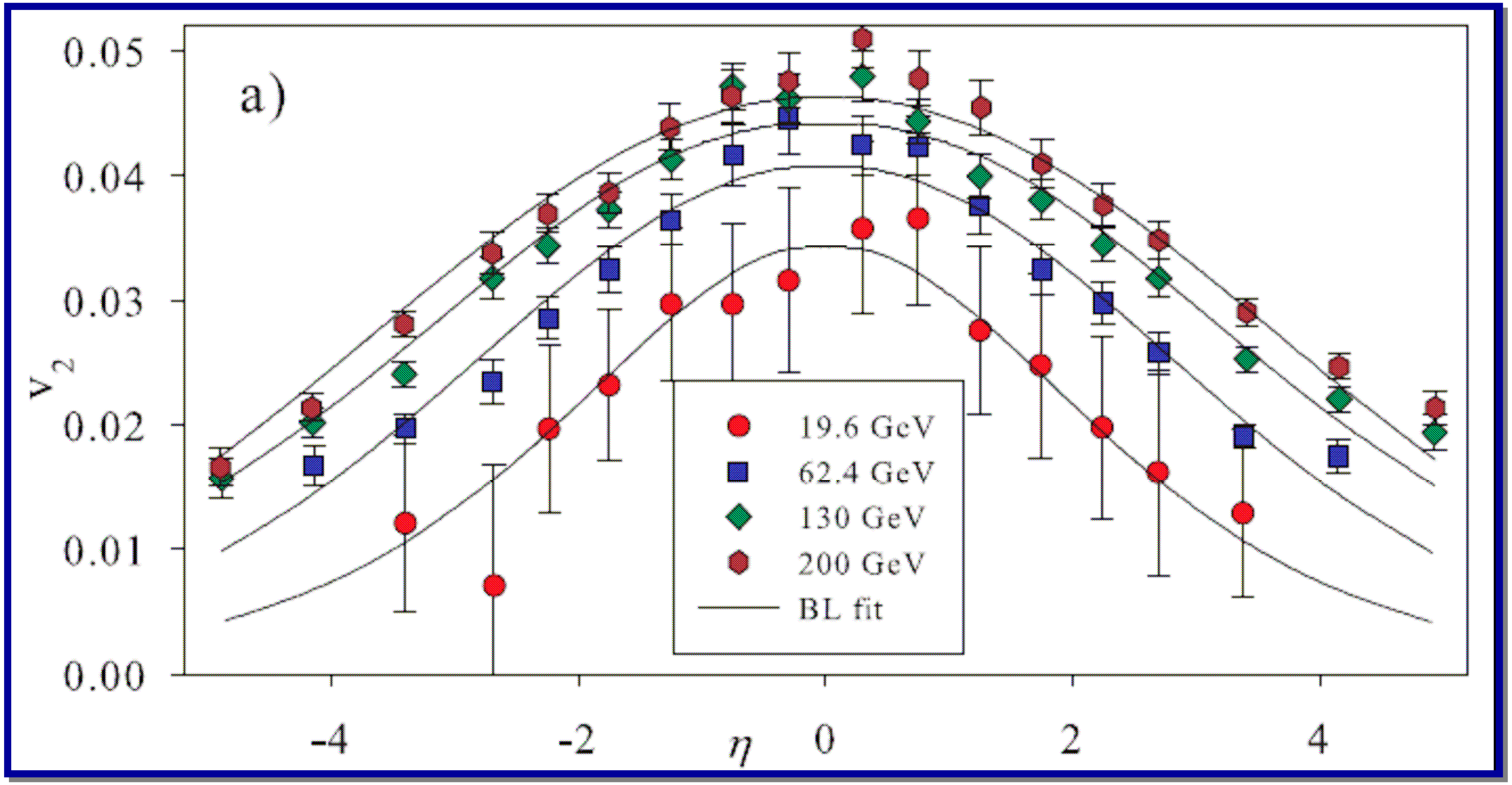
nucl-th/0505019

Scaling reproduced by the Buda-Lund parametrization of the emitting source.

G. Veres, PHOBOS data, proc QM2005
Nucl. Phys. A774 (2006) in press

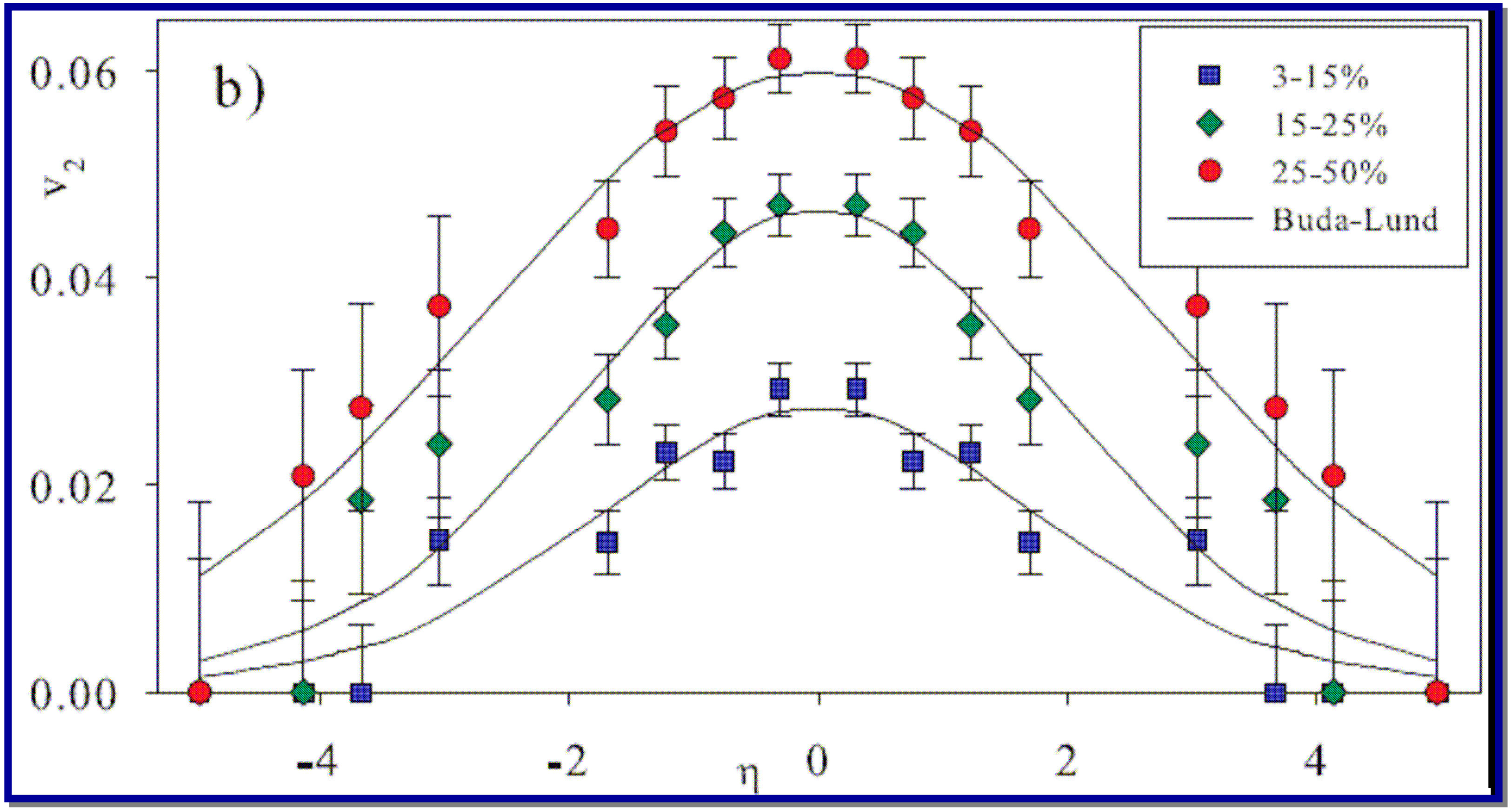
Hydro scaling of v_2 and $\sqrt{s}$ dependence

PHOBOS, nucl-ex/0406021



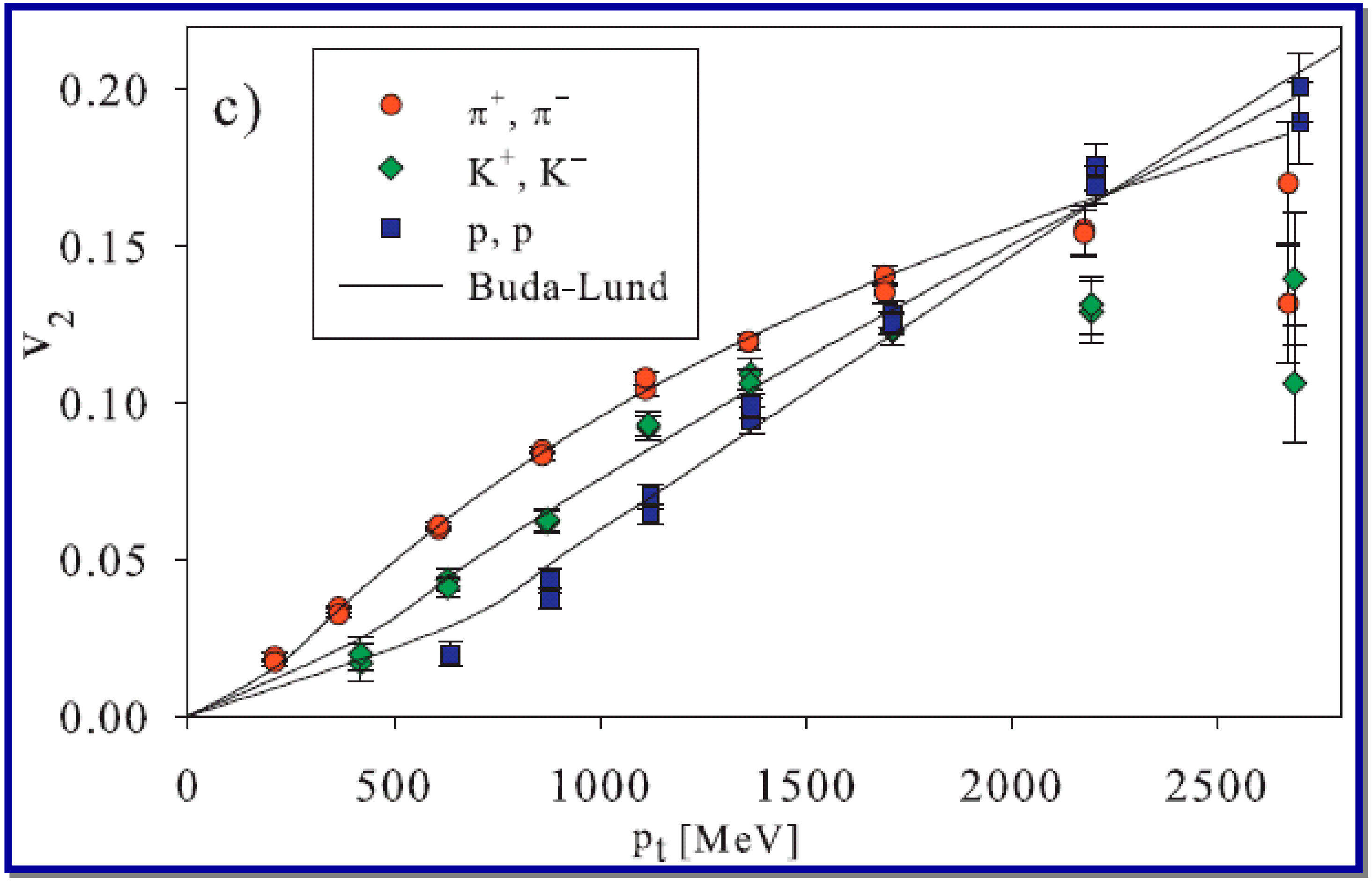
Universal scaling and v_2(centrality,η)

PHOBOS, nucl-ex/0407012



Universal v2 scaling and PID dependence

PHENIX, nucl-ex/0305013



Universal scaling and fine structure of v2

STAR, nucl-ex/0409033